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FATIGUE BEHAVIOR IN STRAIN CYCLING IN THE LOW AND INTERMEDIATE CYCLE RANGE

By S. S. Manson and M. H. Hirschberg (1963) 13p

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio

ABSTRACT

Data are presented on the fatigue characteristics in the life between approximately 10 and 10^6 cycles for a large number of materials including steels, aluminum, titanium, beryllium, and high temperature alloys. Both cyclic strain hardening and strain softening materials were investigated. Linear relationships were found when elastic strain range and the plastic strain range were plotted on log-log coordinates against life. The model selected for representing fatigue characteristics consists of a plot of life versus total strain range which is the sum of elastic and the plastic components. In the low cycle range, the plastic strain range predominates and in the high cycle range the elastic strain range predominates. Empirical relations have been developed for predicting both the elastic and plastic lines from data obtainable in the conventional tensile test. The validity of these predictions is demonstrated by experimental data on a number of alloys. Author

INTRODUCTION

A considerable interest exists at the present time in low cycle fatigue, this interest arising because of the many applications which involve design for finite life. That low cycle fatigue is governed by cyclic plastic strain range has been shown by numerous investigators, and the power law relation between life and plastic strain range as proposed by Manson and Coffin has been amply verified. However, before this relation can be applied it is necessary to determine the cyclic plastic strain range. While in many applications the total strain range may be known, or experimentally determined, separation of the total strain range into its elastic and plastic components may involve some difficulty. For most materials, life above 1000 cycles involves appreciable elastic strain. At about 10,000 cycles the elastic and plastic strain ranges are at about the same order of magnitude, and above 100,000 cycles the plastic strain range is negligible compared to the elastic strain range. In fact, for some very strong materials the elastic strain range may start to predominate at lives of about 100 cycles or less. Thus, although plastic strain range may govern life, the elastic strain range assumes considerable importance in the intermediate cycle life range, either because it is needed in order to compute the plastic strain, or because it is the larger component and may therefore be a better measure of the life than the plastic strain.

Based on limited data available in 1960 it has been proposed (ref. 1b) that the elastic component of the strain range would also satisfy a power law relationship with cyclic life, similar to that existing between plastic strain range and cyclic life. Experimental verification of such relationships were subsequently shown (ref. 2) for sixteen materials of various composition and heat treatment. Thus, it has been fairly extensively verified that life is governed by the total strain range, consisting of an elastic and a plastic component, each of which produces a straight line when plotted against life on log-log coordinates.

Because of this linearity, relatively few experimental data are needed to characterize the fatigue behavior of a material in the low and intermediate cyclic life range between approximately 10 and 10^6 cycles. However, there are applications in which it is desired to estimate the life in advance of fatigue experimentation.

The availability of the large amount of fatigue data in reference 2, and additional data generated since the publication of that report, has made possible the undertaking of an empirical correlation approach for estimating both the elastic and plastic strain range components from static tensile properties alone.

The object of this report is to review the data and verify the previously proposed assumptions that these data can accurately be represented by straight lines in the life range of 10 to 10^6 cycles and to develop a method of predicting these straight lines from simple tensile data.

By comparing the tensile properties of over twenty materials with selected points on their fatigue curves represented in the form of life versus elastic and plastic strain ranges, an approach has been found whereby the static tensile properties can be used to predict the fatigue properties. Checks on the validity of the method are made by comparing the predictions of the method with experimental results for the materials used to obtain the correlation and for six materials tested after the correlation was developed. A comparison is also presented between the experimental results and those predicted by a method recently proposed by Langer (ref. 3), and later studied by Tavernelli and Coffin (ref. 4).

EXPERIMENTALLY DETERMINED STRAIN-CYCLING BEHAVIOR OF MATERIALS

The basic equations of any stress analysis are the equilibrium equations involving stresses, and the compatibility equations involving total strains. Thus, it is the interrelation between the stresses and total strains that is required to solve these equations. For many applications involving cyclic straining, the relation between stress range and total strain range, along with some life relation, will be required for the solution of the problem (ref. 1). Once the stress and strain distributions are determined, it then becomes possible to make some estimate of the cyclic life of the structure. It is found desirable, before deriving the relation between stress range and total strain range, to separate the total strain range into its elastic and plastic components, and to express each of these components in terms of cyclic life.

Relation between plastic strain range and cyclic life.

If a plot is made on logarithmic coordinates of the plastic strain range $\Delta\epsilon_p$ versus the number of cycles to failure N_f, the result is found to be very nearly a straight line. Thus the cyclic life is related to the cyclic plastic strain range by a power law in the form

$$\Delta\epsilon_p = MN_f^z \tag{1}$$

where M and z are material constants.

Equation (1) was first proposed by Manson (ref. 5 and 6) on the basis of limited experimental data by Sachs and his co-workers (ref. 7). The exponent, z, was suggested as a variable, differing among materials. For the aluminum alloy on which the data were available, Manson suggested a value of $z = -1/3$. An improved analysis of the data was later made by

Coffin (ref. 8), who found that $z = -1/2$ provided a better representation of the data, and who also suggested that this value of z is applicable to all materials. In more recent work (ref. 2), the authors found that equation (1) was valid but that z appeared to be a material constant rather than a universal constant. Equation (1), therefore, represents a relation that has been proven valid by a number of investigators, and for a large number of materials.

Relation between elastic strain range and cyclic life.

When fatigue specimens are cycled between fixed strain limits, the stress range generally changes during the test. If the stress range increases with cycles, the material is called a cyclic strain hardening one, and if the stress range decreases with cycles, it is called cyclic strain softening. As was shown in reference 2, the most significant changes in stress range for many materials occurred within the first 20 percent of specimen life. During the remaining 80 percent or more of the life, the stress range remained relatively constant. This value of stress range is then considered as a characteristic value corresponding to the applied strain range. For the purpose of analysis the stress range measured at one half the number of cycles to failure was selected as the characteristic value, and subsequently referred to as the asymptotic stress range $\Delta\sigma$.

For a large number of materials tested, it was found that plots of this stress range (or by dividing it by the elastic modulus and calling it an elastic strain range) versus the cyclic life, on logarithmic coordinates result in reasonably straight lines (refs. 1b and 2). Thus the cyclic life may be assumed to be related to the elastic strain range by a power law in the form

$$\Delta\epsilon_{el} = \Delta\sigma/E = (G/E)\ N_f^{\gamma} \qquad (2)$$

where $\Delta\epsilon_{el}$ is the cyclic elastic strain range corresponding to the cyclic life N_f, E is the elastic modulus, and G and γ are other material constants.

Although equation (2) adequately represents the characteristic behavior of a large number of materials for engineering use, it is admittedly only an approximation of the true material behavior. For practical purposes, however, it can be regarded as valid in the life range of usual interest, up to 10^6 cycles, and in many cases up to even higher lives.

<u>Alternate elastic relation involving an endurance limit.</u>

Equation (2) implies that the life increases with a decrease in elastic strain since the exponent γ is always negative and thus for any elastic strain, corresponding to an applied stress, will predict a finite life. In reality, it is well recognized that many materials exhibit an endurance limit; that is, a stress level below which the life becomes essentially infinite. If it does not become infinite, at least it increases to a greater extent than is implied by equation (2). In order to take cognizance of the possibility of the existence of an endurance limit, the following derivation is made:

Let it be assumed that the asymptotic stress range-strain range relation coincides with the elastic line up to a critical stress range, thus implying that until this stress range is reached, no plastic flow will take place. Since, according to equation (1), finite life occurs only if plastic flow develops, the life will be infinite if the stress range is maintained below this critical level. By definition, then, this stress amplitude then becomes the endurance limit, σ_{end}, and the stress range associated with this critical stress is $2\sigma_{end}$.

For stress ranges above $2\sigma_{end}$ plastic flow occurs, and the life becomes governed by the plastic flow according to equation (1). As a first approximation, let it be assumed that the familiar power law relation exists between the plastic flow and the stress range causing it. That is

$$\Delta\epsilon_p = A(\Delta\sigma - 2\sigma_{end})^d \tag{3}$$

Substituting in equation (3) the value of $\Delta\epsilon_p$ from equation (1), and solving for $\Delta\sigma$

$$\Delta\sigma = 2\sigma_{end} + \left(\frac{\Delta\epsilon_p}{A}\right)^{1/d} = 2\sigma_{end} + \left(\frac{M}{A}\right)^{1/d} N_f^{\,z/d} \tag{4}$$

Or, dividing by the elastic modulus to obtain the relation in terms of strain

$$\Delta\epsilon_{el} = \frac{\Delta\sigma}{E} = \frac{2\sigma_{end}}{E} + \frac{1}{E}\left(\frac{M}{A}\right)^{1/d} N_f^{\,z/d} \tag{5}$$

Equation (5) also relates the elastic strain range as a power law in terms of the cyclic life, but it includes an endurance limit term in contrast to equation (2). In principle, therefore, equation (5) is a much more satisfactory relation, capable of accommodating the concept of an endurance limit. For the numerical purposes associated with many practical problems, it can be shown, however, that the differences involved between the two equations are relatively small.

It is also apparent, from equation (3), that the endurance limit enters directly into the relation between stress range and plastic strain range and this equation might, therefore, also be used directly to determine the endurance limit, without introducing the cyclic life. Thus from equation (3)

$$\Delta\sigma = 2\sigma_{end} + (\Delta\epsilon_p/A)^{1/d} \tag{6}$$

The authors have attempted to determine whether equations (5) or (6) could be used on fatigue data to determine reliable values of endurance limits. Some of the results are shown in the appendix. It was concluded that for even hypothetical sets of data; that is, data with built-in scatter less than that obtained from actual test data, the method requires data at high cyclic life, where plots of $\Delta\epsilon_{el}$ versus N_f show distinct curvature, in order to determine a clearly defined endurance limit. In the absence of such data, equation (2) can be assumed to represent the data adequately in the low and intermediate cycle life range (for most materials up to 10^6 cycles). Because of the simplicity of equation (2), it is used in the remainder of the discussion, with the recognition that it is an approximation that implies the non-existence of an endurance limit (infinite life), but in practice is not inconsistent in representing data in the life range of interest (usually below 10^6 cycles) even for cases involving endurance limits. For materials that demonstrate distinct curvature in plots of $\Delta\epsilon_{el}$ versus N_f at lives well below 10^6 cycles, there is no difficulty in replacing equation (2) by its equivalent, equation (5), wherever the former appears in the discussions to follow.

Relation between total strain range and cyclic life.

Since for many applications it is the total strain range that is of interest, rather than either the plastic or elastic component, equations (1) and (2) can be combined to obtain the desired sum

$$\Delta\epsilon = \Delta\epsilon_p + \frac{\Delta\sigma}{E} = MN_f^{\,z} + \frac{G}{E} N_f^{\,\gamma} \qquad (7)$$

This equation will be used in the remainder of this report as the basic relation between cyclic life and total strain range. Figure 1 shows schematically the graphical implications of equation (7). Plotted against cyclic life on log-log coordinates, both components produce straight lines. Because the strain scale is logarithmic their sum is, however, not a straight line. It can be seen that in the low life range, the elastic component is almost negligible compared to the plastic component. The total strain range, $\Delta\epsilon$, thus almost coincides completely with the straight line for the plastic component. At the higher cyclic lives, however, the plastic strain range rapidly becomes negligible, while the elastic strain range retains a relatively high value because of the lower slope of the $\Delta\epsilon_{el}$ line. Thus, the $\Delta\epsilon$ curve approaches tangency to the elastic line. The cross-over point between the two curves is, for most materials, in the vicinity of 10^4 cycles. Thus, if life ranges of less than 1000 cycles are involved, it is usually permissible to neglect consideration of the elastic component. On the other hand, if the problem involves lives in the vicinity of 100,000 cycles, the strain of major interest is the elastic strain range (or, equivalently, the stress range). Basically, it is probably still localized plastic flow that induces fatigue, even at the very high lives, but measurement of the plastic flow is difficult, and the stress range apparently becomes an adequate measure of this localized plastic flow.

One final point can be made in connection with figure 1 that is of practical interest in the experimental determination of material behavior. It can be seen that if the elastic and plastic lines are to be determined by measurement, the plastic line should be accurately determined in the low-cycle range, where it produces its greatest influence; whereas the elastic line should be most accurately determined in the high-cycle range. Thus, if compromises have to be made in fairing curves to "linearize" them, the range of most significant influence should be favored. This was done in the analysis shown in this report and when analyzing the data in reference 2.

DIRECT DETERMINATION OF STRESS-STRAIN-LIFE RELATIONS

In the discussions to follow, two approaches will be indicated which can be used to determine the inter-relations among stress range, strain range, and cyclic life. As already indicated, the types of relations sought will be those that are based on equation (1) in conjunction with equation (2) instead of the (probably) more realistic equation (5) which includes an endurance limit. More accurate results could be obtained at the expense of additional experimental data. The approach is based entirely on data obtained in fatigue tests.

Measurement of constants.

In the most general sense the constants M, z, G, and γ must be regarded as properties which vary from material to material, and which can best be determined experimentally from strain cycling tests. Although there are four constants they are, in principle, determinable from tests at only two fixed total strain range levels for which the measured quantities are stress range and life. From the stress range, the elastic strain range may be determined, using the elastic modulus. A logarithmic plot of elastic strain range versus life is then constructed from which G and γ can be determined. Subtraction of elastic strain range from total strain range provides the plastic strain range which can be plotted logarithmically against life. Such a plot permits calculation of M and z. If, for example, the applied strain ranges are $\Delta\epsilon_1$ and $\Delta\epsilon_2$, the corresponding measured stress ranges $\Delta\sigma_1$ and $\Delta\sigma_2$, while the cyclic lives are N_1 and N_2 respectively, the constants become

$$\gamma = \frac{\log(\Delta\sigma_1) - \log(\Delta\sigma_2)}{\log N_1 - \log N_2}$$

$$G = \Delta\sigma_1 N_1^{-\gamma} \quad \text{or} \quad \Delta\sigma_2 N_2^{-\gamma} \tag{8}$$

$$z = \frac{\log\left(\Delta\epsilon_1 - \frac{\Delta\sigma_1}{E}\right) - \log\left(\Delta\epsilon_2 - \frac{\Delta\sigma_2}{E}\right)}{\log N_1 - \log N_2}$$

$$M = \left(\Delta\epsilon_1 - \frac{\Delta\sigma_1}{E}\right) N_1^{-z} = \left(\Delta\epsilon_2 - \frac{\Delta\sigma_2}{E}\right) N_2^{-z}$$

Obviously, an improvement in the determination of the constants can be achieved by testing at more than two strain levels. The optimum straight lines are then drawn through the available data points either by inspection or by least square procedures. The constants can then be determined according to equation (8), using any two points on the optimized straight lines, rather than any individual experimental data points.

Also, as already indicated, the most desirable region in which to determine the plastic strain range constants, M and z, is the range of low cyclic life, whereas the most desirable region for determination of the elastic strain range constants, G and γ, is that at high cyclic life. The most accurate determination of the constants will thus occur if the entire life range of interest is covered in the tests. In principle, however, relatively few tests, as low as two, are needed to determine all four constants.

Relation between stress range and strain range.

The principal need in design calculations is for a relation between stress range and strain range. Such a relation provides a means of combining the equilibrium equations involving stresses, and the compatibility equations involving strains. Eliminating N_f between equations (1) and (2), and combining with equation (7), results in

$$\Delta\epsilon = \frac{\Delta\sigma}{E} + M\left(\frac{\Delta\sigma}{G}\right)^{z/\gamma} \tag{9}$$

In equation (9) the stress range is designated $\Delta\sigma$, but it is to be understood that when it is measured experimentally in a cyclic strain range test, during which the stress range may vary continuously, it will be taken as the value at the half-life. Since for most materials and strain ranges, the stress range has essentially stabilized at a constant value by the time the half-life is reached, the stress range $\Delta\sigma$ has already been referred to as the asymptotic stress range. Thus equation (9) represents essentially a relation between the total strain range and the asymptotic stress range.

It should be pointed out, moreover, that what is desired for analysis is a relation between stress range and total strain range in order to permit inter-relation between the equilibrium and compatibility equations. It is not necessary that the relation be expressible analytically, as in equation (9), although the equation appears suitable for this purpose for most of the materials examined to date. The relation could be expressed completely graphically, as by a curve best passing through the data as is usually done for a static stress-strain curve. No limitations are imposed in the numerical analysis of specific problems by the use of a graphical relation instead of an analytical expression.

SOME APPROXIMATE RELATIONS

Although it is relatively simple to measure the cyclic stress strain relation, and the life characteristics under strain cycling, specimens and laboratory facilities are not always available to obtain the necessary data. Especially in the early stages of design, it is desirable to be able to make estimates based on available data of the most limited type. Thus approximate relations involving readily available data can serve a very useful purpose, provided the limitations of these approximations are recognized. In the following sections some approximate relations will be developed that may serve adequately in preliminary analysis. For the final analysis of a chosen design, it would appear reasonable to expect that the actual properties of the material involved will be determined by test rather than relying on approximations.

The data used for determining the predicting parameters are shown in figures 2 through 17 and were taken from some earlier work of the authors (ref. 2) except that data points with cyclic lives less than 10 cycles were omitted from these plots. The approximate relations to be developed are those that will best predict or represent the least square elastic and plastic strains versus life lines (on log-log plots) for the wide variety of materials tested. The least square lines used are the "A" lines of figures 2 through 17. In calculating these lines, the following procedure was used: for the plastic line, the data points used were those for which the plastic strain range was greater than half the elastic strain range, while for the elastic line the data points used were those for which the elastic strain range was greater than half the plastic strain range. This was done, as was pointed out earlier, in order to obtain the best fit of the data in the regions where they are most significant.

It can be seen from figs. 7 and 14 that these two materials (annealed 304 and AM 350 steels) gave very nonlinear trends in the plastic data due to the fact that they are both basically unstable and transform during cycling. Thus, at each strain level, or life, a somewhat different material is being tested. For this reason, these two materials (nos. 6 and 13) were omitted from plots made for the purpose of determining the parameters for predicting the plastic strains. The static tensile properties to be used in the following analysis are listed in table I.

<u>Parameters</u>. - Before deriving any relations, it will be instructive to consider first the properties that enter into the approximations, and the basis for their use.

<u>Tensile ductility</u>. - The tensile ductility is a property usually available to the designer for any material likely to be of interest.

Since the ductility is a plastic strain value, it would appear desirable to make use of it in the plastic strain range-life equation (1). In the discussion to follow ductility will be taken as the "true" or "logarithmic" value, based on measurement of reduction in area in the tensile test. Thus

$$D = \ln \frac{A_o}{A_f} = -\ln (1 - R.A.) \tag{13}$$

where D is the ductility, A_o and A_f are the initial and final areas of the fracture cross-section in the tensile test, and R.A. the conventional reduction in area $= \frac{A_o - A_f}{A_o}$

Coffin (ref. 8) has proposed that the ductility be introduced into the plastic strain range-life equation in such a manner that the plastic strain range becomes equal to the ductility when the number of cycles is equal to 1/4. The probable reasoning behind this suggestion stems from the concept that a conventional tensile test constitutes one quarter of a cyclic strain test in which the strain is completely reversed. If the quarter-cycle constituting the tensile test were carried to completion as part of a cyclic strain test, the load would first be reduced to zero in the second quarter of the cycle, a compressive force applied during the third quarter causing a compressive plastic flow, and a return to zero load during the fourth quarter, placing the specimen in a position for repetition of the strain cycle.

The manner of including the tensile test as an extreme case of cyclic plastic flow is, however, open to some question. Another form of reasoning has been offered by Martin (ref. 9) who treated the problem of cumulative fatigue damage, and who arrived at a form of equation (1) such that the effective value of $\Delta\epsilon_p$ is $\sqrt{2}\,D$ at $N_f = 1/4$, while Manson (ref. 1c) arrived at another effective value of $\Delta\epsilon_p$ of 1.5 D at $N_f = 1/4$ cycle.

The problem of including the conventional tensile test as an extreme of the cyclic test is, actually, only an academic question. In reality, the tensile test is quite different from the cyclic straining test. Thus, the validity of considering the tensile test as a quarter, or some other fraction of cyclic straining test would depend on the experimental determination of the consequences of the assumption, rather than on its theoretical significance. It is therefore important to examine how well this assumption is borne out for specific materials as a means of establishing its validity. Figure 18 shows the relation between $(\Delta\epsilon_p/D)_{1/4}$ versus D for the materials being investigated. The data were obtained by determining the intercept of the least square plastic lines at $N_f = 1/4$ (figures 2A through 17A). If the assumption that $\Delta\epsilon_p = D$ at $N_f = 1/4$ were valid, all the points would lie on a horizontal straight line $(\Delta\epsilon_p/D)_{1/4} = 1$. Considerable deviation is seen to exist between this assumption and the data. A detailed analysis of the intercept of the least square plastic lines and other values of N_f was tried. It was found that a much improved correlation could be obtained if the intercept with the life of 10 cycles were used. This correlation is shown graphically in figure 19 and the curve shown representing the data is

$$(\Delta\epsilon_p)_{10} = \frac{1}{4}\,D^{3/4} \tag{14}$$

On the basis of this figure it can be seen that the ductility is most useful in determining one point on the plastic line, and that point can be taken at 10 cycles.

Ultimate tensile strength. - The ultimate tensile strength is defined as the maximum load sustained by a specimen during a tensile test, divided by the original cross-sectional area. For most ductile materials the maximum load occurs after appreciable elongation (and reduction in area). This property must thus be regarded as highly artificial, since the load and area used in the computation do not occur simultaneously. However, it is the most commonly cited property used as a measure of material strength; hence it is desirable to determine whatever correlations that can be obtained as a guide in estimating fatigue properties.

It has long been known that the ultimate tensile strength can be used to give some indication of the endurance limit of a material. This type of correlation has not always been successful, but it does indicate that this property is related in some way to the behavior at high cyclic life. Plots were therefore made of the intercept of the least square elastic lines at a number of different cyclic lives (figs. 2A through 17A) versus ultimate tensile strength for all the materials investigated. The best correlation obtained was at 10^5 cycles as is shown in figure (20). It can be seen that as a reasonable first approximation

$$(\Delta\sigma)_{10^5} = 0.90\ \sigma\mu \qquad (15)$$

or

$$(\Delta\epsilon_{el})_{10^5} = \frac{(\Delta\,\sigma)_{10^5}}{E} = 0.90\,\frac{\sigma\mu}{E} \qquad (16)$$

While this relation must obviously be regarded as very approximate - since it relates two different types of tests by a property that is not even realistically related to either test -- it serves the very useful purpose of indicating the approximate location of one point on the straight line represented by equation (2). Thus, one point on the line of elastic strain versus cyclic life can be determined by considering the life at 10^5 cycles. The elastic strain range developed at this life is approximately 90 percent of the elastic strain developed by a stress equal to the (nominal) ultimate tensile strength of the material.

Tensile fracture stress. - The fracture stress is determined by dividing the load just prior to fracture by the area measured just after fracture. Although the load decreases after the ultimate tensile stress is reached, the cross-sectional area decreases more rapidly, thus resulting in a progressively increasing "true stress". In all cases, therefore, the fracture stress is either equal to or greater than the ultimate tensile stress.

Even though the fracture stress takes into account actual areas, and is therefore a "true" stress, it is still a somewhat idealized property because it does not take into account triaxiality and non-uniformity of stress that develops in a tensile specimen after "necking" takes place. In view of the artificiality of this property, together with the fact that it is obtained in a static tensile test which does not involve the material in either the cyclic hardening or softening that develops in a fatigue test, it can be expected that the utility of this property in predicting the fatigue behavior will be limited. Since, however, only approximations are sought, a correlation was attempted for the large number of materials investigated both in tension and in axial fatigue.

If the tensile test is to be regarded as one-quarter cycle of a fatigue test, it is natural to expect that the line of elastic strain (or corresponding stress) associated with equation (2) intersect the life $N = 1/4$ at a stress range which is twice the fracture stress. Figure 21 shows the correlation obtained when the fracture stress is plotted against the intercept at $N_f = 1/4$ of the optimized linear relation between elastic strain and cyclic life on log-log coordinates. The relation is very nearly linear, indicating that

$$(\Delta\epsilon_{el})_{1/4} = \left(\frac{\Delta\sigma}{E}\right)_{1/4} = 2.5\,\frac{\sigma_f}{E} \tag{17}$$

where σ_f is the fracture stress in the uniaxial tensile test.

Thus equation (17) provides information from which a point on the elastic line represented by equation (2) can be determined. It is merely necessary to plot $2.5\,\frac{\sigma_f}{E}$ at $N_f = 1/4$. This relation, together with equation (16) provides sufficient information to determine the elastic line of equation (2).

There is, however, a difficulty that may arise in trying to use σ_f as a predicting parameter and that occurs when dealing with highly ductile materials. For such materials, it becomes almost impossible to get a good measure of actual load carrying area at the time of failure, and hence σ_f, because of the many cracks known to be present throughout the highly necked down section. For these extremely ductile materials it is best not to attempt to use σ_f for predicting a point on the elastic line, but rather to use some average slope for the line passing through the elastic strain range predicted at 10^5 cycles from σ_u. The average slope recommended for these cases is -0.1 as will be discussed later.

<u>Approximate relationship of total strain at 10^4 cycles.</u> - Thus far three relations have been indicated for the determination of the two straight lines associated with the elastic and plastic component of strain; only one more relation is required for the complete establishment of these two lines. The relation that has been found most useful is based on figure 22 which shows a plot of longitudinal total strain range versus number of cycles to failure for the large number of materials which data are available (ref. 2). It is interesting to observe that all the curves (except for beryllium) seem to come together at approximately 10^4 cycles, and at a total strain range of approximately 1 percent. Thus, regardless of material, a total strain range, consisting of the sum of elastic and plastic strain ranges, of approximately 1 percent will result in a life of 10,000 cycles. Actually, the relation that is most useful arises out of a refinement of this observation. A plot of $\Delta\epsilon_{el}$ versus $\Delta\epsilon_p$ at 10^4 cycles is shown in figure 23. A straight line does represent the data fairly well, but the equation of the line is

$$(\Delta\epsilon_p)_{10^4} = .0069 - 0.52(\Delta\epsilon_{el})_{10^4} \tag{18}$$

instead of

$$(\Delta\epsilon_p)_{10^4} = 0.01 - (\Delta\epsilon_{el})_{10^4}$$

which would result if the best relation were represented by the sum of the two strains being equal to 0.01.

Equation (18) thus represents a usable relation for the determination of the plastic strain range at 10^4 cycles when the elastic strain range is known. Since by equation (2) the elastic strain range versus cyclic life is approximated by a straight line on log-log coordinates, and since two points on this line can be determined from equations (16) and (17), equation (18) is adequate for determination of the plastic strain range required to cause fracture at 10^4 cycles.

There is a possible difficulty that may arise in using equation (18) and that is when dealing with very high strength materials where the predicted value of elastic strain range at 10^4 cycles approaches or is greater than 0.0132. When this happens, the error in computing $\Delta\epsilon_p$ from equation (18) can be very great. For such cases when the computed value of $\Delta\epsilon_p$ is less than 0.001 it is felt that some average slope of the plastic line through the predicted point at 10 cycles should give more reasonable results. The average slope recommended for these very high strength materials is -0.6 as will be discussed later.

Endurance limit. - The most common definition of the endurance limit is the stress at the outermost fibers in an alternating bending test below which failure does not occur regardless of how many cycles are applied. In practice, however, the endurance limit is taken as a specific point on the $\Delta\sigma$ - N_f fatigue curve of a material; for steels the point is frequently at 10^6 cycles. Choice of an arbitrary life is necessary not only because of the practical difficulties of determining precisely the "knee" of the $\Delta\sigma - N_f$ curve, but also because some materials do not have well defined "knees". Failure occurs at almost any stress level if the number of cycles of stress application is great enough. Thus, information on the endurance limit makes available one point on the $\Delta\sigma - N_f$ curve, either by implication as to cycles to failure, or by direct specification of cyclic life. Thus if σ_{end} is the endurance limit at a life of N_{end} cycles, one point on the line of elastic strain range versus life is $\frac{2\sigma_{end}}{E}$ at $N = N_{end}$, where E is the elastic modulus.

It should be recognized, however, that the endurance limit specified as conventional engineering information frequently refers to data obtained in alternating bending tests, whereas the discussion here refers to axial straining. For the present it may be recognized that since only gross approximations are desired, the two types of endurance limits may be used interchangeably by noting from reference 10 that

$$\sigma_{end}^{ten} = 0.65\ \sigma_{end}^{bend} \tag{19}$$

In the analysis to be discussed the uniaxial endurance limit is used in two different methods. The first method makes use of the assumption that the line represented by equation (2) is horizontal, that is $\gamma = 0$. Thus the elastic strain range is a constant over the entire life range, and since it is known at one value of life, it is known at all values. Thus

$$\Delta\epsilon_{el} = \frac{G}{E} N_f^{\,0} = \frac{G}{E} = \frac{2\sigma_{end}}{E}$$

or

$$G = 2\sigma_{end} \tag{20}$$

In the second method the point at the "knee" of the endurance curve is used instead of the relation involving the ultimate tensile strength, equation (16). The "knee", for numerical purposes to be discussed, is assumed at 10^7 cycles. Thus, instead of equation (16), use is made of the relation

$$\left(\frac{\Delta\sigma}{E}\right)_{10^7} = \frac{2\sigma_{end}}{E} \tag{21}$$

Equation (21) is then combined with equation (17) to determine the constants in equation (2). Since the determination of the endurance limit involves considerable fatigue testing, and since the purpose of the approximations to be discussed herein is to obtain life estimates from the most readily determined mechanical properties, the relations involving endurance limit must be regarded as secondary to those involving properties determined from the tensile test alone.

Constant slope values. - Since the purpose of the approximate formulas to be derived is to obtain only estimates of cyclic life, it may sometimes be sufficient to use slope values of both the elastic and plastic strain range components as determined from other materials tested under suitable conditions. For the plastic component of strain, a slope of -1/2 has been suggested by Coffin (ref. 8). Figure 24 shows a plot of the best fit slopes (the same as the exponent z in eq. (1)) versus the ductility for the materials of reference 2. It is seen that most materials have negative slopes of greater magnitude than -1/2. A better "average" value might be -0.6 if it were desirable to use the same value for all materials. It is this "average" slope value that was recommended for use when trying to predict the behavior of very high strength materials as was previously discussed.

The slope of the line fitting the elastic component of the total strain range (the same as the exponent γ in eq. (2)) was found to range from -0.06 to -0.16 among the materials analyzed. Where no other information is available, an average value of slope $\gamma = -0.1$ may be assumed, but no use is made of this simplification in this report except for the cases of very ductile materials where σ_f cannot be measured accurately.

Relation involving ductility and endurance limit.

An extremely simple relation was proposed by Langer (ref. 3), which relates the total strain range and cyclic life where the following assumptions were made regarding the previously discussed parameters.

a) The plastic strain range is equal to the ductility at a cyclic life of 1/4 cycle,

b) The plastic exponent is taken as -1/2 for all materials,

c) The elastic strain component is constant, and is taken as the elastic range at the endurance limit.

Under these conditions the resulting equation for total strain range becomes

$$\Delta\epsilon = \Delta\epsilon_p + \Delta\epsilon_{el} = \frac{D}{2}\,(N_f)^{-1/2} + \frac{2\sigma_{end}}{E} \qquad (22)$$

Relation involving ductility, ultimate tensile strength and fracture stress.

The two lines constituting the elastic and plastic components of strain range can be determined using the tensile data relations involved in equations (14), (16), (17), and (18). The sum of the two components then yields the total strain range in terms of cyclic life and properties determined from the uniaxial tensile test. In practice a graphical procedure proves to be very simple. The line for the elastic component is constructed first by establishing the strain range at 1/4 cycle and 10^5 cycles from the fracture stress and ultimate tensile stress according to equations (16) and (17) or by passing a line of slope -0.1 through the calculated strain at 10^5 cycles for materials where σ_f cannot be measured. The elastic strain range at 10^4 cycles is then read from the predicted straight line. From the elastic strain range the plastic strain at 10^4 cycles is determined using equation (18). The point at 10^4 cycles is then joined by a straight line to the point at 10 cycles determined by the ductility using equation (14). For the case where $\Delta\epsilon_p$ at 10^4 is computed to be less than 0.001, a line of constant slope -0.6 is passed through the 10 cycle point. The ordinates at selected values of cyclic life are then added to give total strain range.

It is, however, possible to perform the steps analytically providing relations for M, z, G, and γ in terms of D, σ_μ and σ_f. These relations become

$$\Delta\epsilon = \Delta\epsilon_{el} + \Delta\epsilon_p = \frac{G}{E} N_f^{\gamma} + M N_f^{z} \qquad (23)$$

where

$$G = \frac{9}{4}\,\sigma_\mu \left(\frac{\sigma_f}{\sigma_\mu}\right)^{0.9} \tag{24}$$

$$\gamma = -\,0.083 - 0.166 \log \left(\frac{\sigma_f}{\sigma_\mu}\right) \tag{25}$$

$$M = 0.827D \left[1 - 82 \left(\frac{\sigma_\mu}{E}\right)\left(\frac{\sigma_f}{\sigma_\mu}\right)^{0.179}\right]^{-1/3} \tag{26}$$

$$z = -\,0.52 - \frac{1}{4} \log D + \frac{1}{3} \log \left[1 - 82 \left(\frac{\sigma_\mu}{E}\right)\left(\frac{\sigma_f}{\sigma_\mu}\right)^{0.179}\right] \tag{27}$$

Relation involving ductility, fracture stress and endurance limit.

If an endurance limit is available it is, of course, preferable to use a fatigue property to establish the elastic strain range relation instead of resorting entirely to the properties from the uniaxial tensile test. In this case it is logical to construct the line for elastic strain range by using the point at the known endurance limit together with the point at 1/4 cycle determined by the fracture stress. If the endurance limit is given at 10^7 cycles, use is then made of equation (21) together with equation (17) to construct the elastic strain range line. However, it should be recognized that the "endurance limit" as considered here is regarded as a point on the straight line of strain range (or stress range) versus cyclic life. For materials in which the elastic curve tends to level off considerably, so that a quoted endurance limit is beyond the effective "knee" of the curve, it is obvious that use of the specified endurance limit will yield inaccuracies in the construction of the elastic line. Hence, caution should be used in applying quoted endurance limits unless the life at the endurance limit is also specified, and it is reasonably certain that the point given occurs at the "knee" or before it in magnitude of cyclic life.

Graphically, the procedure for using an endurance limit is identical to that described previously, except that the point on the elastic line at 10^5 cycles determined from the ultimate tensile strength is replaced by the point at the endurance limit. Analytically, the problem is slightly more complicated because the life at which the endurance limit is taken, N_{end}, must be left as an assignable variable. The formulas for G, γ, M, and z become fairly complicated if N_{end} is included as a literal term. It is thus desirable to derive separate formulas for specific values of N_{end}. Those below refer to a value of $N_{end} = 10^7$

$$G = 2.5 \ \sigma_{end} \left(\frac{\sigma_f}{\sigma_{end}}\right)^{0.92} \tag{28}$$

$$\gamma = - 0.013 - 0.13 \log \left(\frac{\sigma_f}{\sigma_{end}}\right) \tag{29}$$

$$M = 0.827D \left[1 - 166 \left(\frac{\sigma_{end}}{E}\right) \left(\frac{\sigma_f}{\sigma_{end}}\right)^{0.394}\right]^{-1/3} \tag{30}$$

$$z = 0.052 - \frac{1}{4} \log D + \frac{1}{3} \log \left[1 - 166 \left(\frac{\sigma_{end}}{E}\right) \left(\frac{\sigma_f}{\sigma_{end}}\right)^{0.394}\right] \tag{31}$$

RESULTS AND DISCUSSION

The availability of experimental data on a relatively large number of materials makes possible a check of the validity of the proposed relations over a broad range of the variables. It should be recognized, of course, that since the relations were derived, in part, from the same data used to check their validity, there exists a bias toward the correlation which cannot be resolved without further data on additional materials. Since these correlations were arrived at in 1961 (ref. 11) the authors have tested 6 additional materials. These materials were not used in obtaining the correlations but are included in this report for the purpose of checking the predicting methods. The tensile data for these materials are listed in table I and the experimental fatigue data are plotted in figures 25 through 30.

Comparisons of predictions with experimental data for two methods are presented along with the least squares, or best fit curves, for the 22 materials investigated. These comparisons are given in figures 2 through 17 and 25 through 30. The "C" lines are the predictions based on the ductility and endurance limit as described by equation (22). In assigning a value of endurance limit an extrapolation of the elastic strain range data to 10^7 cycles was used and these values are listed in table I. The "B" lines of these figures represent the predictions by equations (23) through (27) where only the properties obtained from the uniaxial tensile test as listed in table I were used. It can be seen that in general, equation (22) yields conservative values of life for a given total strain range, while the use of equation (23) in conjunction with the constants of equations (24) to (27) yield life values that more closely comply with the data.

A more complete comparison of the two methods is shown in figures 31(a) and 31(b). Each of these figures shows the ratio of predicted total strain range to the experimentally determined value against cyclic life for all the materials investigated. For these figures the experimentally determined total strain range was taken as the sum of the least squares lines for the elastic and plastic components. Thus the ratios could be taken at all values of life without regard for specific values at which data were obtained.

In figure 31b the predictions are based on equation (22), using an experimentally determined ductility and endurance limit. The endurance limits used to obtain figure 31b were not directly determined, but were rather as previously mentioned obtained by extrapolation of the total strain range to a life of 10^7 cycles. Thus the method as evaluated here is given the benefit of an accurate measure of endurance limit (for the purpose of correlating the lower life data). In general it is seen that this method yields conservative values of strain for a given life. Where conservative design is desirable, the method may serve very well, but it must be recognized that for some materials and in some life ranges the allowable strain predicted by this method will be as low as 1/4 the actual value. In addition, the method requires the experimental determination of an endurance limit in order to correlate the long-life data at all.

The predictions of figure 31a are based on making use of the ductility, fracture stress, and ultimate tensile strength as determined in the static uniaxial tensile test. An improvement is obtained relative to the correlation of figure 31a, although for a given life the predicted strain is sometimes higher and sometimes lower than the measured strain, whereas in figure 31b the predicted strain is generally lower. It is possible to make the method using tensile data alone predominantly conservative by dividing the predicted strain by approximately 1.5; the result is still an improvement (in the sense that better correlation is obtained) over the method using equation (22), despite the fact that no fatigue properties are required to make the analysis.

A final point to be made in comparing the two methods is the very important by-product resulting from the method that fits the elastic strain range data best as well as the total strain range data. This enables the designer to get a first approximation to the stress range - strain range curve which can be used in a stress analysis to obtain a better approximation to the total strains in a structure than if an elastic analysis alone were made. This improved value of total strain range would then result in an even better estimate of life. The prediction based on a horizontal elastic line through the endurance limit results in an inaccurate representation of the stress-range - strain range data and hence can only be used for estimating life from the total strain range, but it cannot aid in the computation of this value.

CONCLUSIONS

The following conclusions are based upon extensive analysis of room temperature strain-cycling fatigue data for the twenty-two materials presented in this paper.

1) The elastic and plastic components of total strain range versus life data measured in the life range of 10 to 10^6 cycles can adequately be represented by straight lines on log-log coordinates, for most materials investigated. The only exceptions were the plastic component of those materials that are unstable and transform during cycling.

2) A method was presented which attempts to determine a clearly defined endurance limit from low and intermediate cycle fatigue data. It was concluded that unless the elastic strain range versus life curve shows distinct curvature in this region, no such clearly defined endurance limit can be obtained and therefore the simple linear relation which adequately represents the data can be used.

3) A simple method for predicting the fatigue behavior of materials from their uniaxial tensile properties is presented. Predictions based upon this method as well as the method of Langer were compared with the data for a large number of materials. The results indicate that the proposed method is in general an improvement over the Langer method which has an added disadvantage of requiring an endurance limit. The proposed method gives a very satisfactory representation of the total strain range versus life relation from 10 to 10^6 cycles and has an added advantage in that it also predicts the stress range-strain range relation which is useful in the analysis of any cyclicly loaded structure.

APPENDIX

Some computations involving estimation of endurance limit.

Equations 5 and 6 are both of the form $y = a + bx^n$. In practice, experimental data are available for corresponding values of y and x, and the problem is to determine the best values of a, b, and n which will correlate the data according to this equation. There are several methods available to do this (ref. 12) but unfortunately none involves a direct plot of y versus x on some coordinate system which permits the optimum choice of the constants. The method that was therefore used by the authors is as follows:

a) select a value of the exponent n,

b) plot y versus x^n,

c) determine by conventional least squares method the values of a and b resulting from the best fit straight line through the data,

d) determine the suitability of the choice of exponent n by calculating the "standard deviation" (ref. 13), which is a measure of the average deviation of the data points from the optimum straight line,

e) repeat the previous 4 steps for a sequence of selected values of n.

Among the various values of n chosen, that value which yields the lowest "standard deviation" can be regarded as the best value. Initially, the spacing between values of n chosen can be quite coarse, but as the best choice is narrowed down, the spacing can be chosen as fine as desired to obtain the best value of n and the associated best values of a and b. Although these computations can be performed manually, the availability of high speed computing machinery greatly reduces the amount of labor and does not discourage refinements in computation by choice of closely spaced values of the exponent n.

Figure 32 shows property curves for a hypothetical material. The solid lines are idealizations of material properties where the endurance limit is taken as zero. Thus, the $\Delta\sigma = \Delta\epsilon$ curve shows plastic flow at all stress levels (although the deviation from the elastic line is very small in the vicinity of the origin) while the $\Delta\epsilon_{el} - N_f$ line is perfectly straight. Equations for these curves are also given in figure 32. The circles represent hypothetical "data" points, and fit the assumed equations exactly. The question to be answered in this illustration is whether, given the hypothetical data points shown by the circles, the proper endurance limit (in this case zero) will unambiguously be indicated by the analysis.

Table II shows the results of the computation performed by the method described for determining the endurance limit from the $\Delta\epsilon_{el} - N_f$ curve as seen in fig. 32b. The assumed values of the exponent z/d in equation (5) are shown in column 1 of Table II. For each assumed value of z/d, equation (5) results in a simple straight line of $\Delta\sigma/E$ versus $N_f^{z/d}$. Using standard statistical methods, the "least squares" straight line was obtained for each assumed value of z/d, the standard deviation of the points from the line is indicated in column 2. The standard deviation is, of course, zero for the value of $z/d = -0.085$, since this exponent is the one on which the hypothetical points are based, but it can be seen that the standard deviation is quite small for even considerably erroneous values of z/d. Each "erroneous" value of z/d produces an "indicated endurance limit", column 4, which compensates for the error in the choice of z/d, and results in a curve representing equation (5) that is in close agreement with the data points.

The dotted lines of figure 32(b) indicate the agreement between the various equations resulting from the least squares fits, and the "data" points on which they are based. In the range of the "data" points, in this case between 10 and 10^5 cycles, it is clear that the choice of optimum fit is not completely unambiguous. Of course, the "data" here were tailored to give an exact value of endurance limit of zero, but small deviations in the "data", so characteristic in fatigue experiments, could easily make the determination of the endurance limit by this method quite ambiguous.

Table III shows similar computations using the cyclic stress-strain characteristic of figure 32(a) as the basis for determining the endurance limit. As before, best results are obtained for $\sigma_{end} \approx 0$, but the standard deviations are small for other choices of $1/d$, and corresponding endurance limits. The degree of fit between the "data" and the various curves representing other values of $1/d$ are shown in figure 32(a). No difference can be detected in these curves for the scale used to plot them.

Further calculations to elucidate the problem are shown in tables IV and V and figure 33. In this case the material is assumed to show an endurance limit of 50,000 psi. The governing equations are assumed to be

$$\Delta\sigma = 100{,}000 + 284{,}000 \, (\Delta\epsilon_p)^{0.246} \tag{32}$$

and

$$\Delta\sigma = E\epsilon_{el} = 100{,}000 + 300{,}000 \, (N_f)^{-0.160} \tag{33}$$

where $E = 32.3 \times 10^6$ psi. However, for the present calculation cognizance will be taken of scatter normally characteristic of fatigue data by arbitrarily displacing the "data" points from the basic equations (32) and (33). The displacements range between ±1 percent to ±5 percent, and the exact magnitudes were chosen by use of tables of random numbers. The "data" points are shown in figure 33 by the circles, and the basic curves (32) and (33) by the continuous lines. The curves and "data" in these computations are shown in figure 33 and tables IV and V. In this case, the "data" are limited to cyclic lives of 10^5 cycles. By comparing the standard deviations in tables IV and V and the dotted curves in figure 33 (of which only two are shown, to avoid congestion), it can be seen that considerable ambiguity exists at the optimum endurance limit. The "data" can be fitted well by curves which vary considerably in endurance limit.

A final computation is shown in figure 34. The data for the range up to 10^5 cycles are here identical to those shown in figure 33 and additional "data" points are included to extend the range to 10^8 cycles. The computations are shown in table VI. Here it can be seen that the ambiguity of endurance limit determination is greatly reduced. Thus, if high cycle data are available, the endurance limit can be determined by the method outlined, but if only low cycle data are available, the method does not accurately determine the endurance limit.

Although the principles involved and the conclusions of the computations described above were illustrated by the use of hypothetical data, the author and his co-workers have attempted the procedure on data for numerous materials which were determined experimentally. The conclusions drawn were approximately the same: the method requires high-cycle data, or data in the range where $\Delta\epsilon_{el}$ versus N_f show distinct curvature, in order to determine a clearly defined endurance limit. In the absence of high cycle data, an equation in the form of (2) can adequately represent the data in the low cycle range (for most materials, up to 10^6 cycles). Because of the simplicity of equation (2) it is used in the body of this paper, with the recognition that it is an approximation that <u>implies</u> the non-existence of an endurance limit (infinite life), but in practice is not inconsistent in representing data in the life range of interest (usually 10^6 cycles) even for cases involving endurance limits. For materials that demonstrate distinct curvature at lives well below 10^6 cycles, there is no difficulty in replacing equation (2) by its equivalent (5).

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TABLE I. - MATERIAL PROPERTIES

NUMBER	MATERIAL	YOUNG'S MODULUS, $\times 10^{-6}$	POISSON'S RATIO	TRUE DUCTILITY	FRACTURE STRESS, KSI	ULTIMATE STRESS, KSI	ENDURANCE STRESS, KSI
1	4130 SOFT	32.0	0.290	1.120	245.0	130.0	45.0
2	304 HARD	25.0	.340	1.165	295.0	138.0	40.0
3	4340 HARD	29.0	.300	.477	278.0	213.0	55.0
4	4340 ANNEALED	28.0	.320	.570	174.0	120.0	50.0
5	52100	30.0	.290	.119	323.0	292.0	80.0
6	304 ANNEALED	27.0	.270	1.368	278.0	108.0	40.0
7	4130 HARD	29.0	.280	.792	302.0	207.0	70.0
8	AISI 310	28.0	.300	1.006	197.0	93.3	18.0
9	INCONEL X	31.0	.310	.223	219.0	176.0	55.0
10	TITANIUM 6 Al-4VA	17.0	.330	.530	249.0	179.0	70.0
11	BERYLLIUM	42.0	.024	.017	47.7	46.9	24.0
12	AM 350 HARD	26.0	.300	.233	328.0	276.0	90.0
13	AM 350 ANNEALED	28.0	.320	.737	339.0	191.0	55.0
14	1100 ALUMINUM	10.0	.330	2.090		16.2	5.0
15	5456 H311 ALUMINUM	10.0	.330	.424	81.7	57.8	20.0
16	2014 T6 ALUMINUM	10.0	.330	.288	91.4	73.6	25.0
17	4130 X-HARD	30.0	.280	.619	352.8	247.8	80.0
18	52100 X-HARD	28.0	.290	.011	366.6	362.6	115.0
19	TITANIUM 5Al-2.5 Sn	16.8	.310	.566	190.0	130.4	60.0
20	VASCOMAX 300 CVM	27.0	.300	.707	380.3	295.1	115.0
21	2024 T4 ALUMINUM	10.0	.300	.402	103.2	71.8	20.0
22	7075 T6 ALUMINUM	10.0	.320	.327	121.2	89.9	20.0

DESCRIPTION OF MATERIALS NOT LISTED IN REFERENCE 2

NUMBER	MATERIAL	NOMINAL COMPOSITION, PERCENT	CONDITION	HARDNESS
17	AISI 4130 (EXTRA HARD)	SAME HEAT AS IN REF. 2	1600° F; 1/2 HR IN ARGON, WATER QUENCH 400° F; 1 HR, AIR COOL	RC 48
18	AISI 52100 (EXTRA HARD)	SAME HEAT AS IN REF. 2	1520° F; 1/2 HR IN ARGON, OIL QUENCH 400° F, 2 HR, AIR COOL	RC 61-62
19	TITANIUM (5Al, 2.5 Sn)	C 0.022, F_e 0.08, N_2 0.014, Al 5.1, Sn 2.5, O_2 0.067, H_2 0.0096, Ti REMAINDER	ANNEALED BY SUPPLIER	RC 31-32
20	VASCOMAX 300 CVM	C 0.03, Si 0.01, Mn 0.02, S 0.0065, P 0.004, Mo 5.00, Co 8.94, Ni 18.51, Ti 0.56, Al 0.05, Zr 0.008, B 0.0012, Ca 0.02, Fe REMAINDER	SOLUTION ANNEALED BY SUPPLIER 900° F; 1 HR, AIR COOL	RC 54-55
21	2024 T4 ALUMINUM	AS PER NAVY SPECIFICATION QQA-268 CONDITION T	AS RECEIVED	RB 94
22	7075 T6 ALUMINUM	AS PER NAVY SPECIFICATION QQA-282 CONDITION T	AS RECEIVED	RB 79

CS-28938

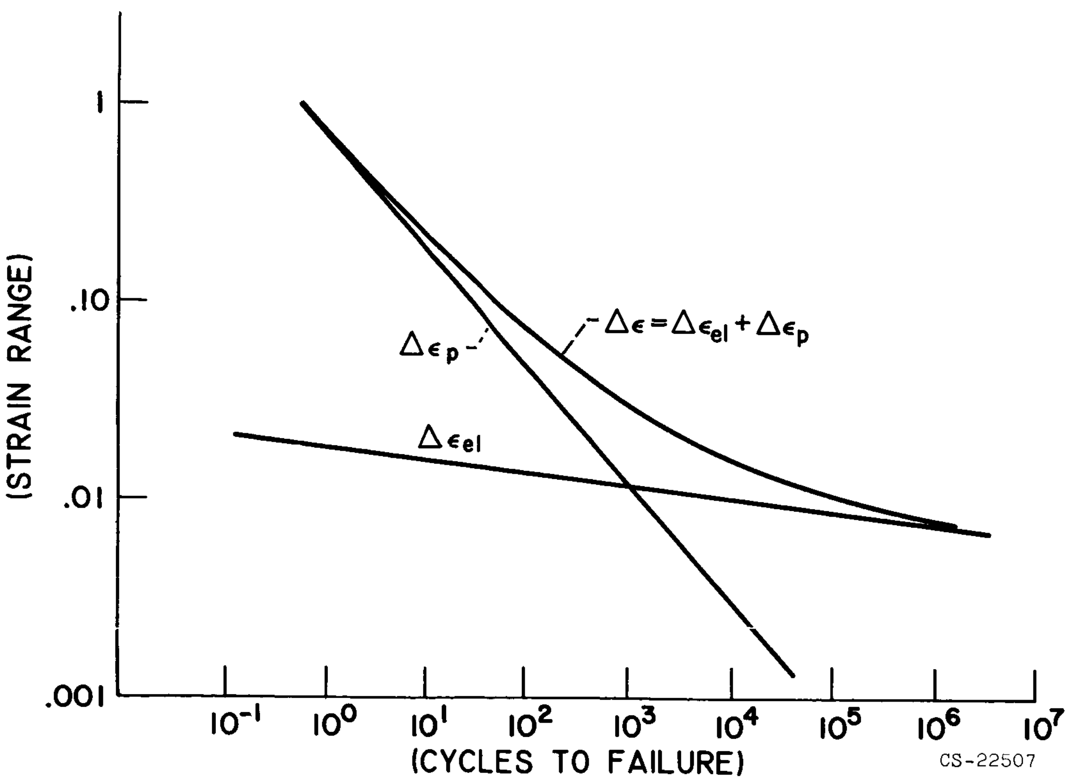

Figure 1. - Total strain range as the sum of elastic and plastic components.

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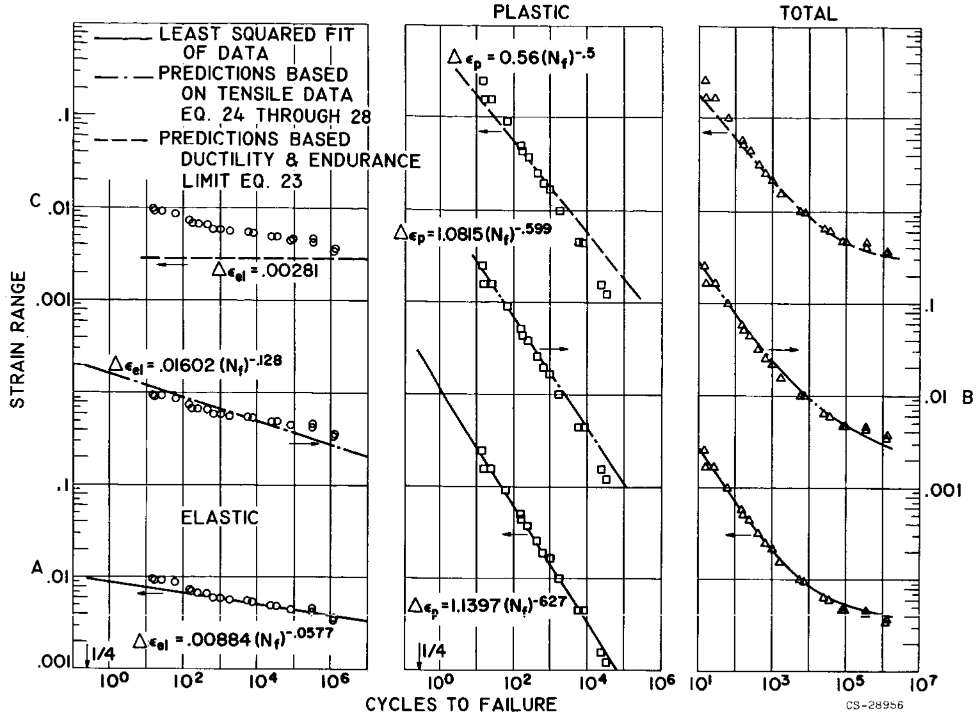

Figure 2. - Fatigue behavior of AISI 4130 soft (material number 1) - see Table I for material identification.

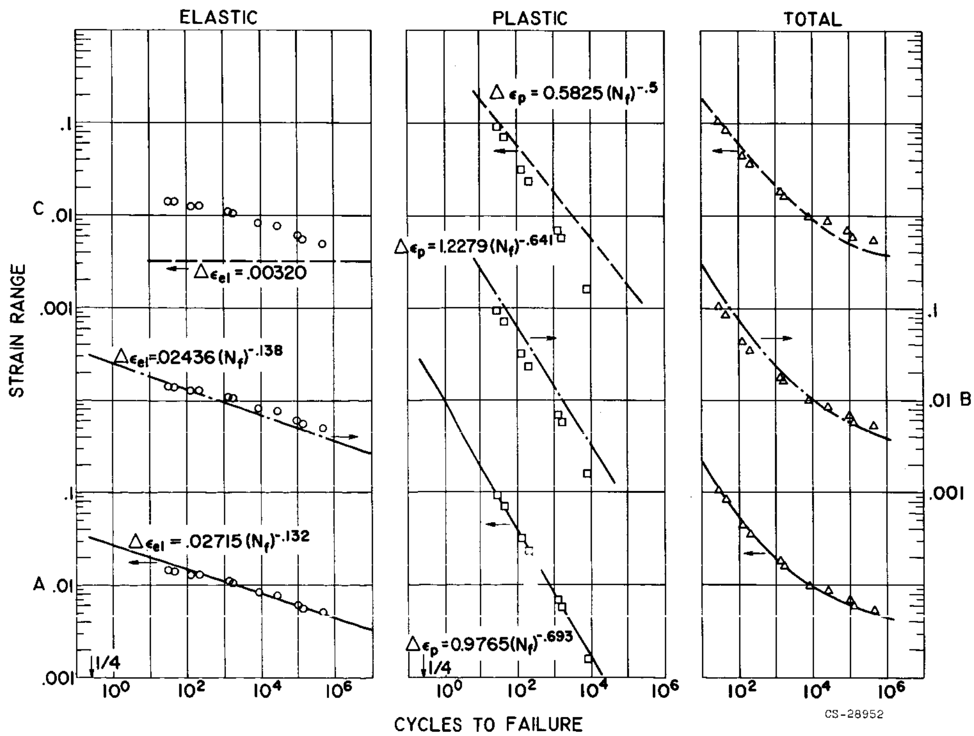

Figure 3. - Fatigue behavior of AISI 304 (hard), material number 2.

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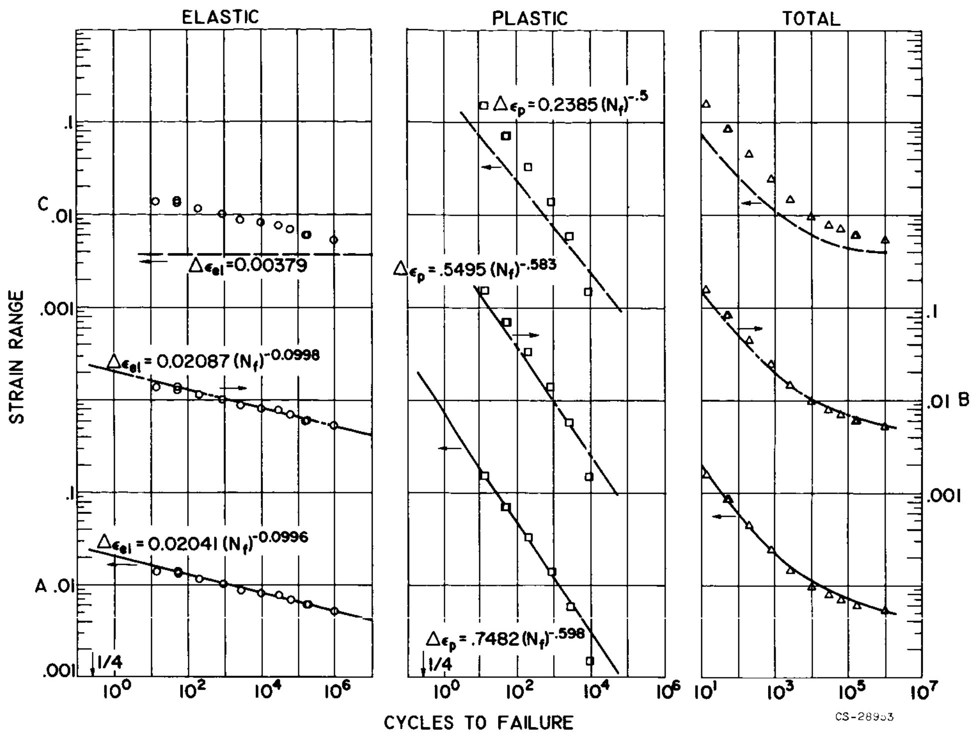

Figure 4. - Fatigue behavior of AISI 4340 (hard), material number 3.

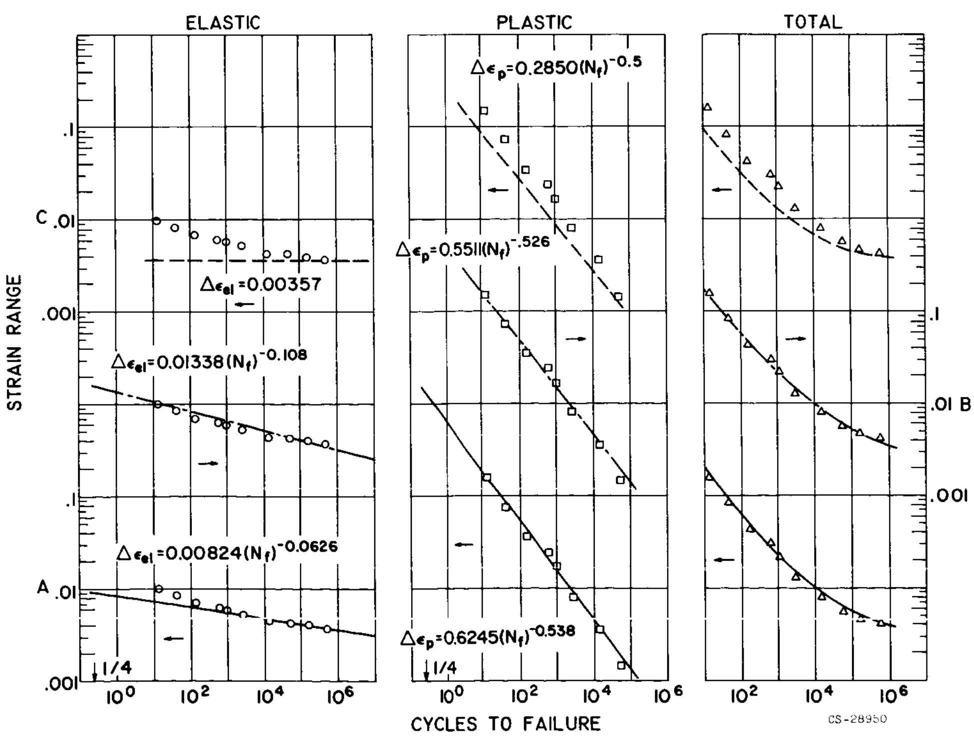

Figure 5. - Fatigue behavior of AISI 4340 (annealed), material number 4.

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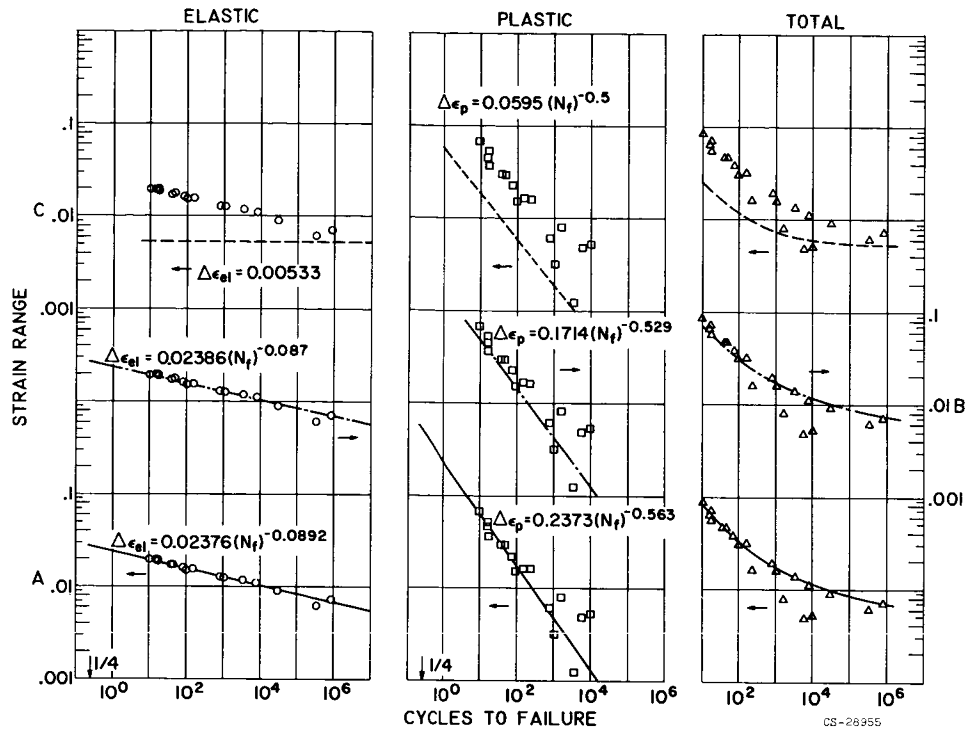

Figure 6. - Fatigue behavior of AISI 52100, material number 5.

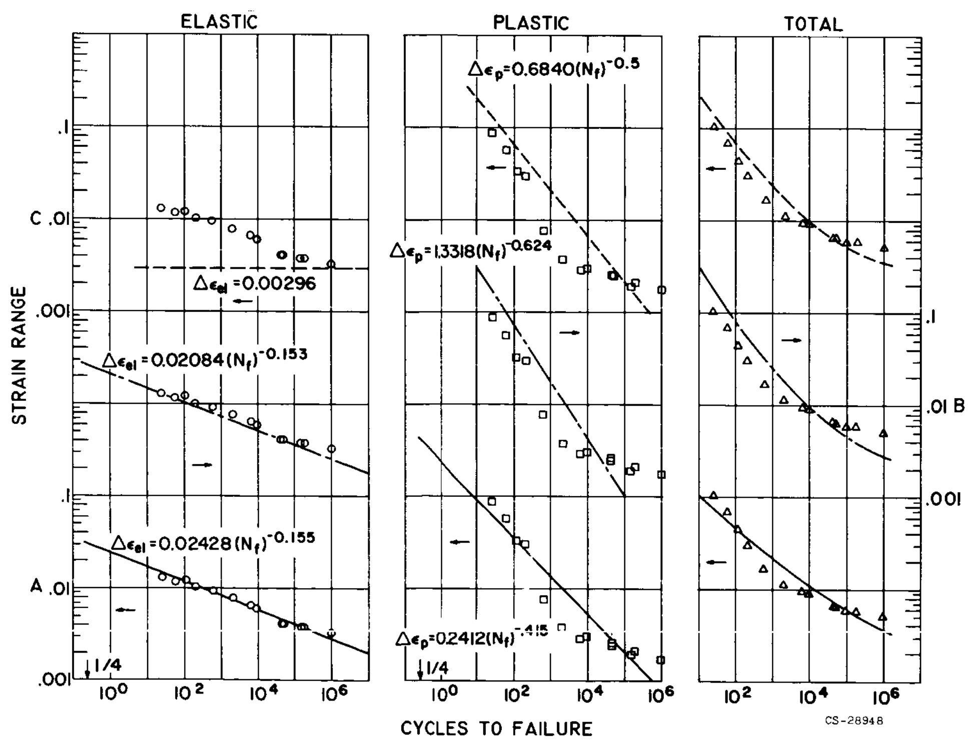

Figure 7. - Fatigue behavior of 304 ELC (annealed), material number 6.

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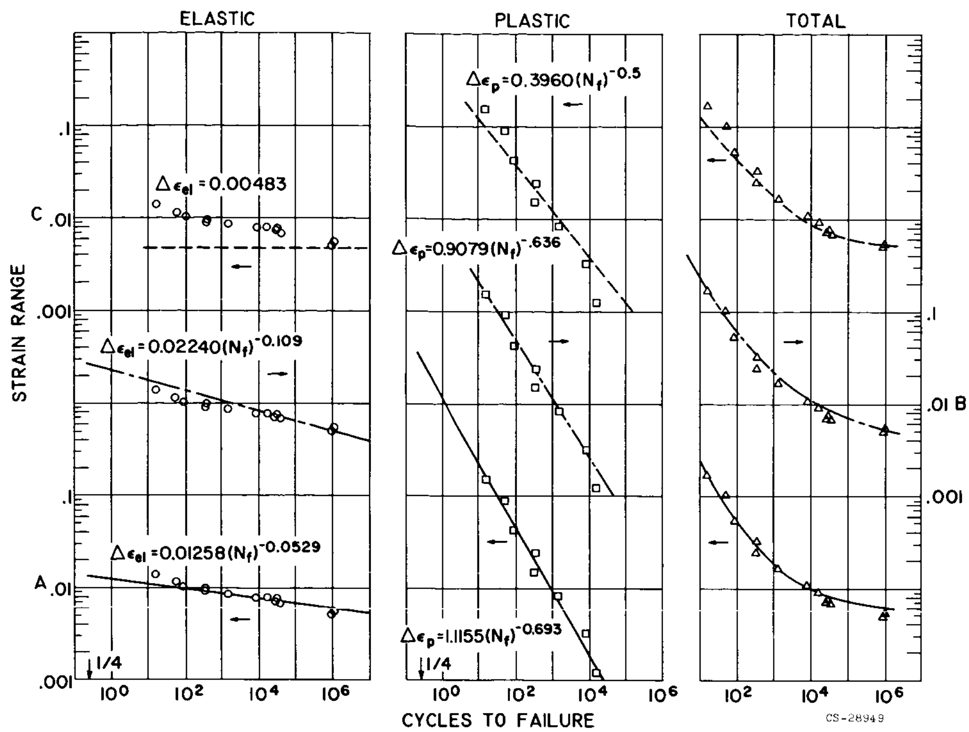

Figure 8. - Fatigue behavior of AISI 4130 (hard), material number 7.

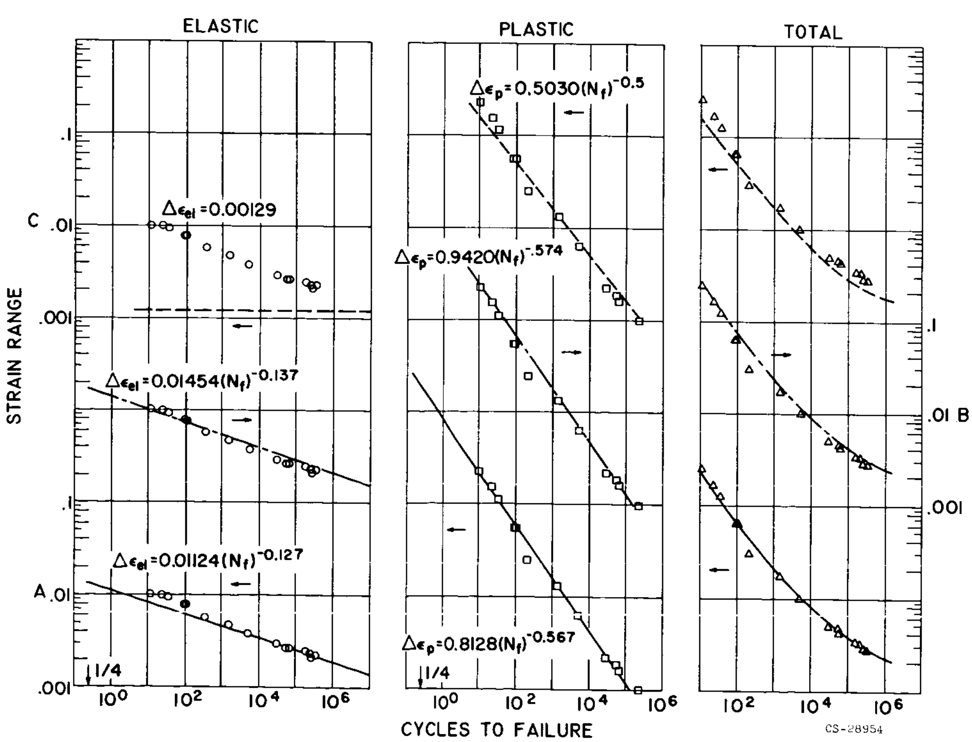

Figure 9. - Fatigue behavior of AISI 310 (annealed), material number 8.

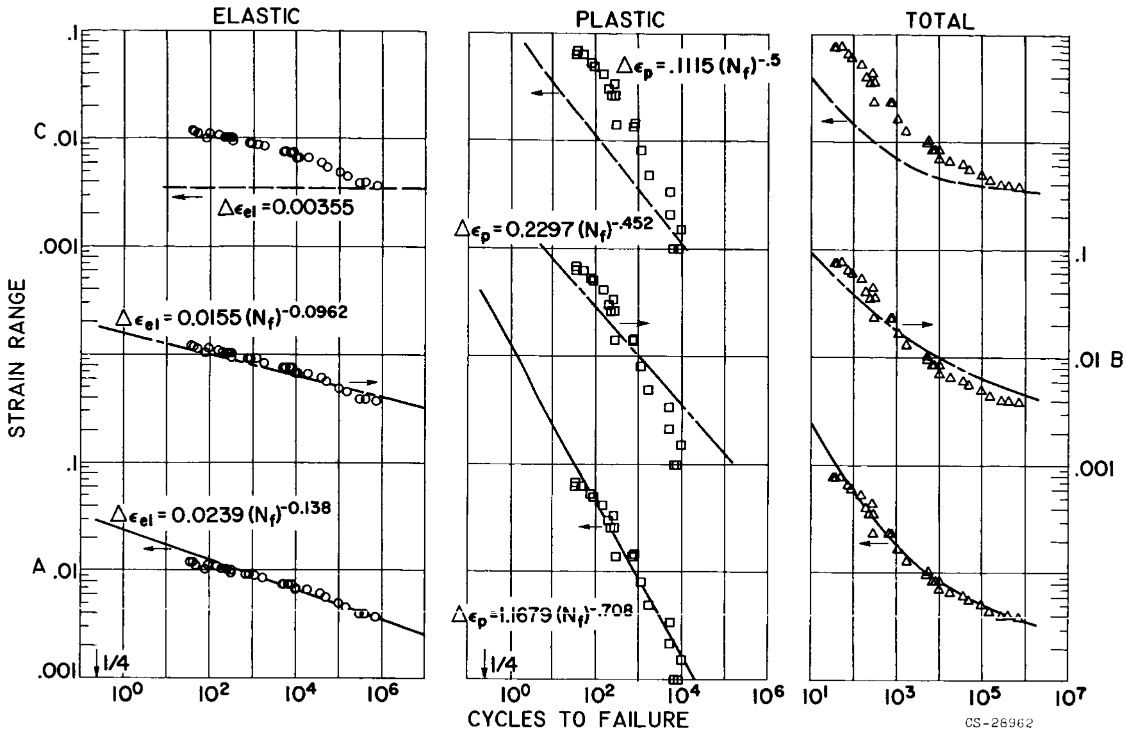

Figure 10. - Fatigue behavior of Inconel X, material number 9.

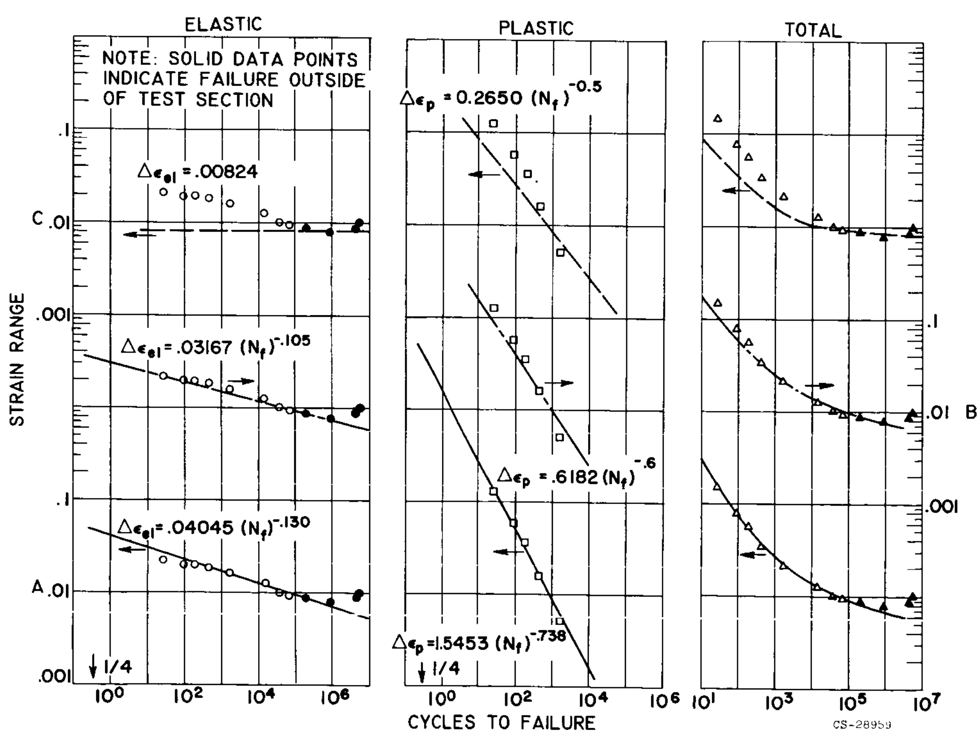

Figure 11. - Fatigue behavior of titanium (6Al, 4Va), material number 10.

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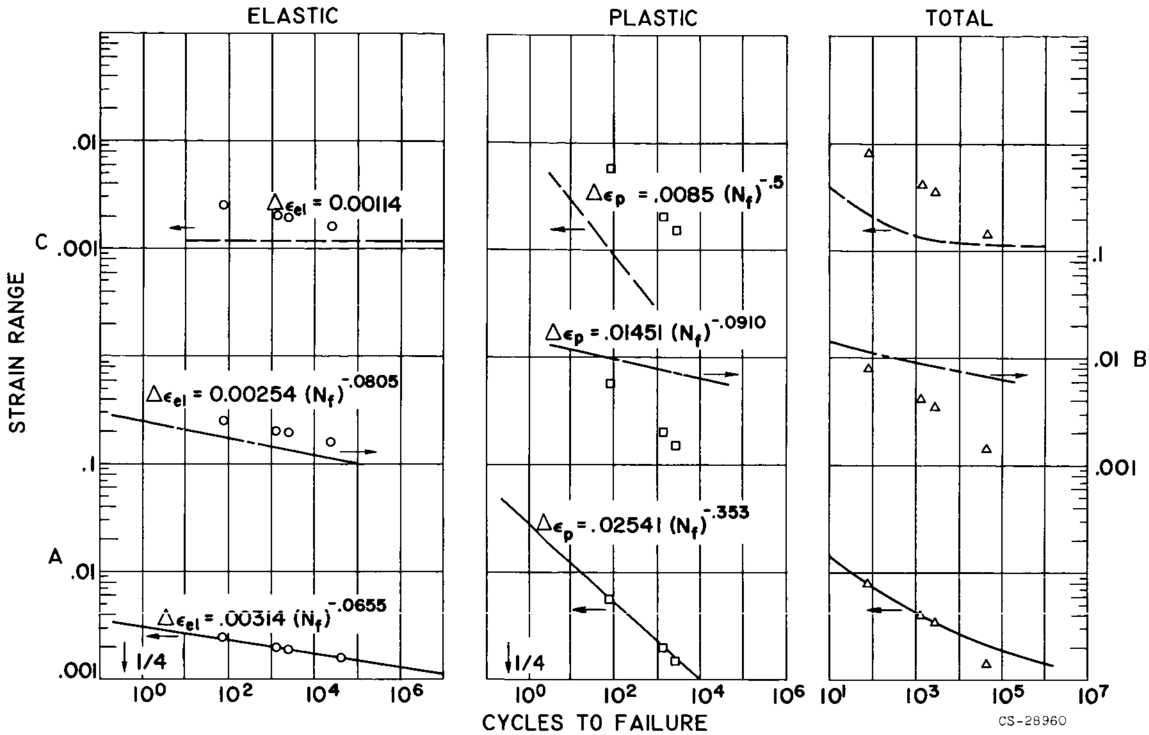

Figure 12. - Fatigue behavior of beryllium, material number 11.

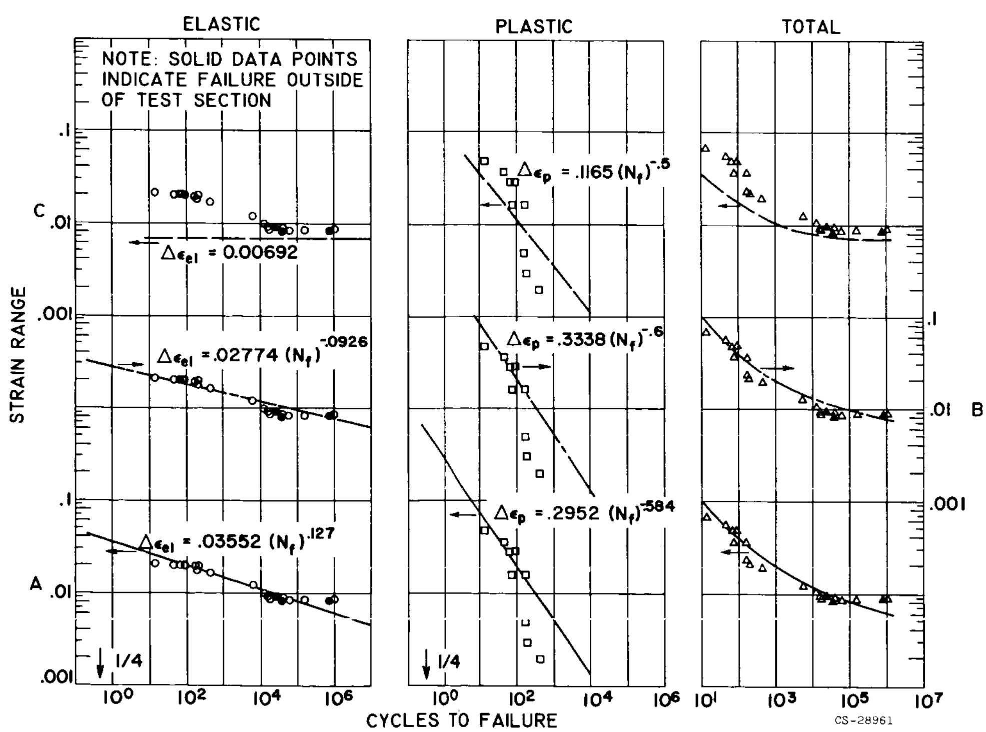

Figure 13. - Fatigue behavior of 350 (hard), material number 12.

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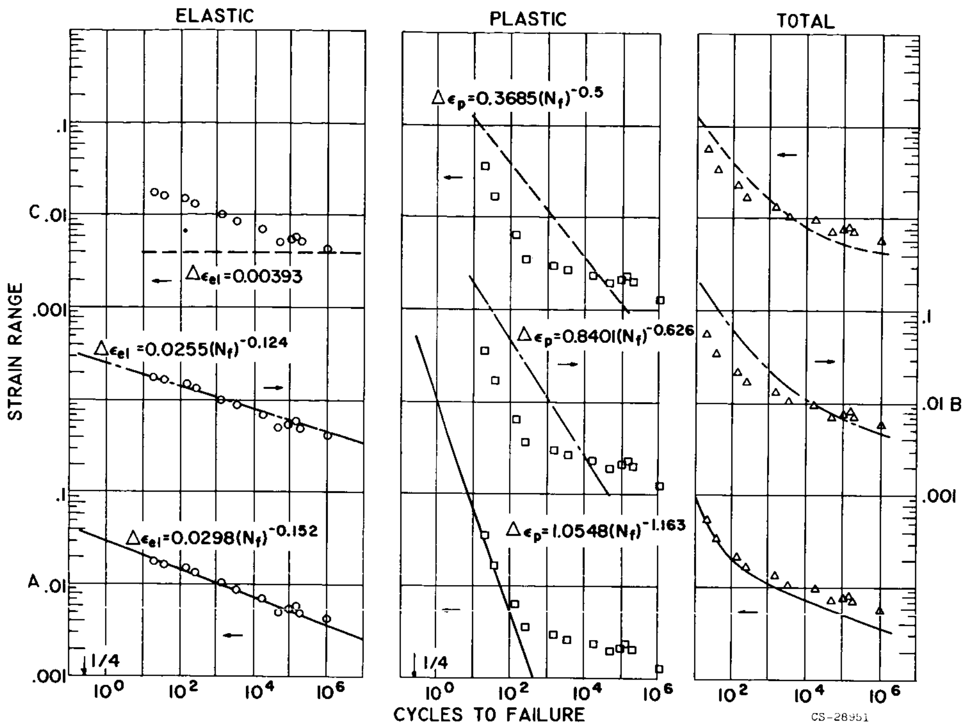


Figure 14. - Fatigue behavior of 350 (annealed), material number 13.

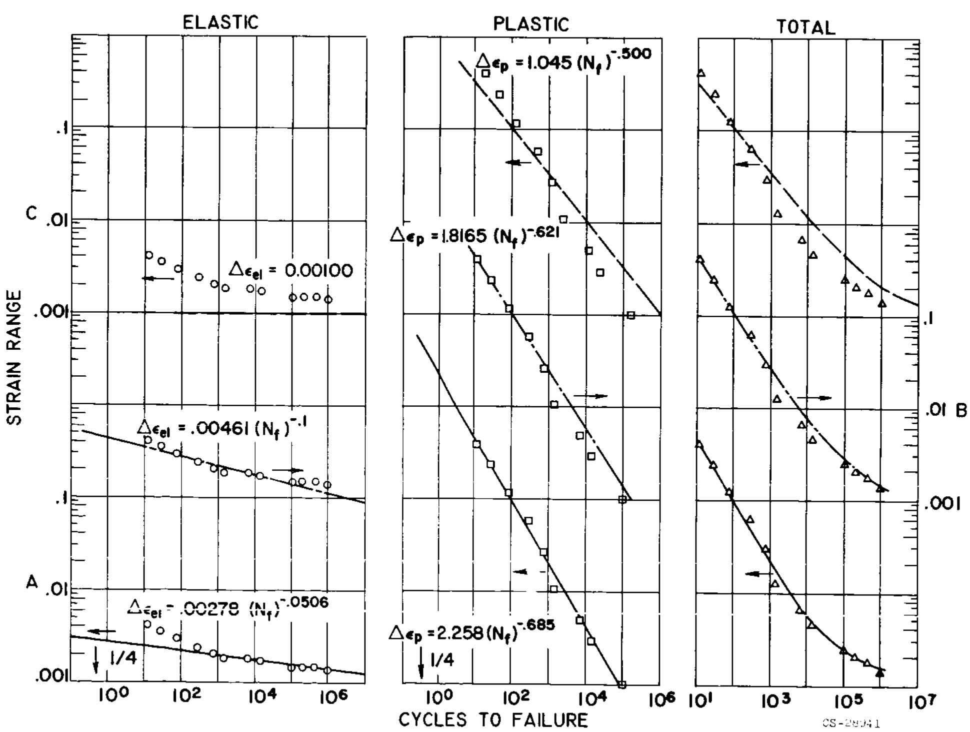


Figure 15. - Fatigue behavior of 1100 aluminum, material number 14.

E-2256

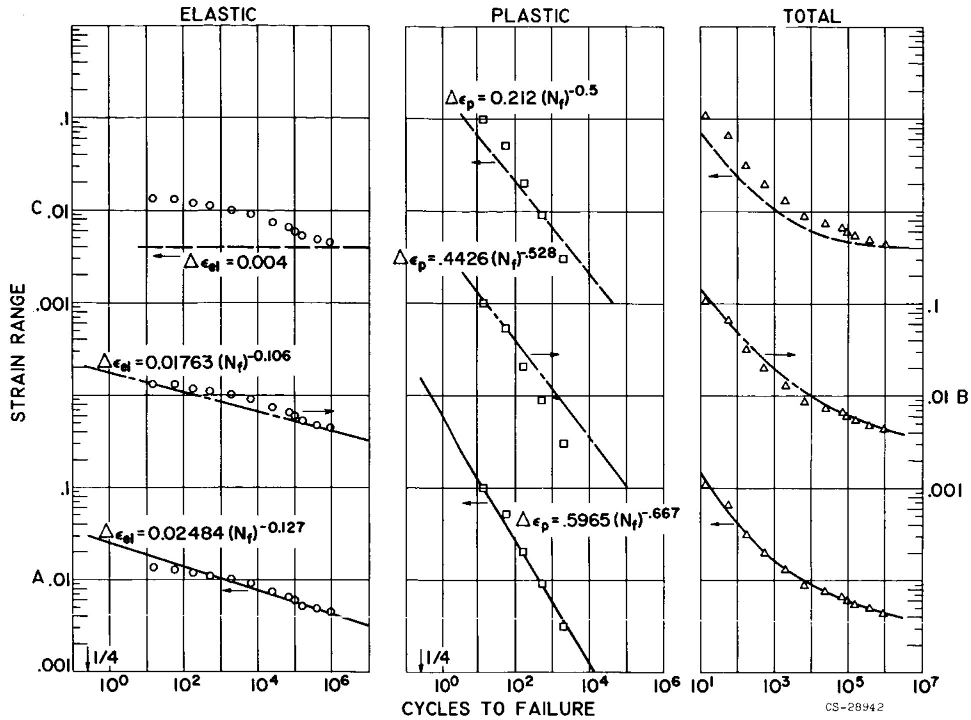


Figure 16. - Fatigue behavior of 5456 H311 aluminum, material number 15.

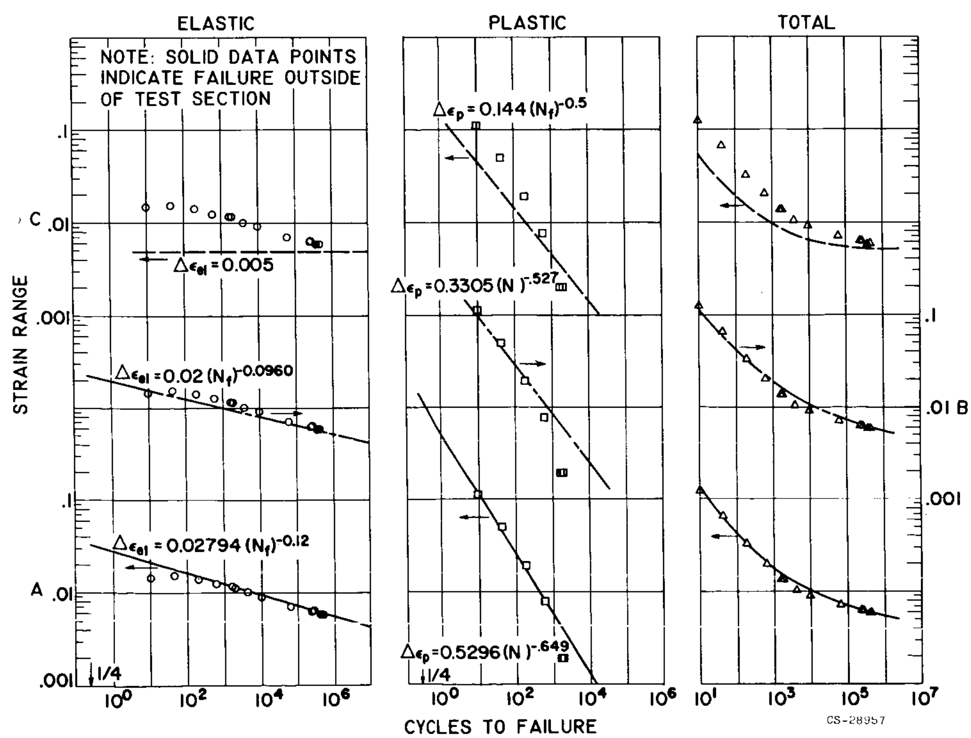


Figure 17. - Fatigue behavior of 2014 T6 aluminum, material number 16.

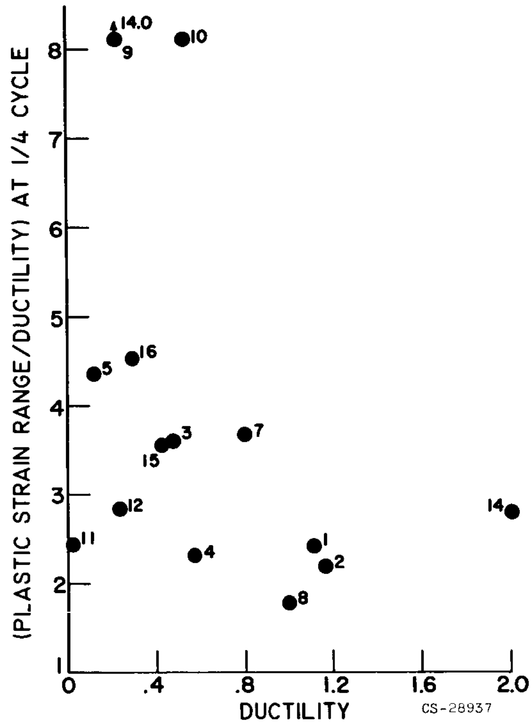


Figure 18. - Intercept of plastic line at 1/4 cycle life as function of ductility. See Table I for material identification.

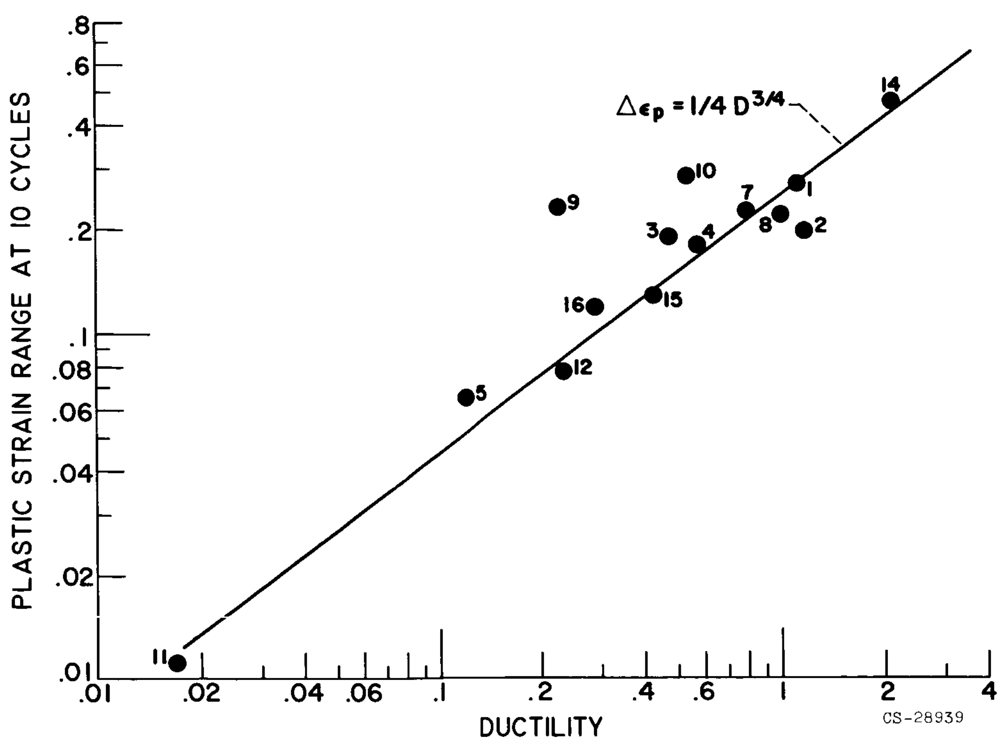
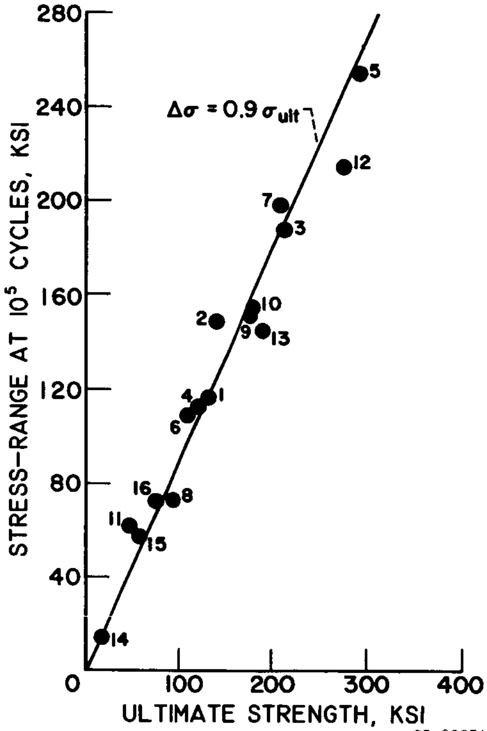
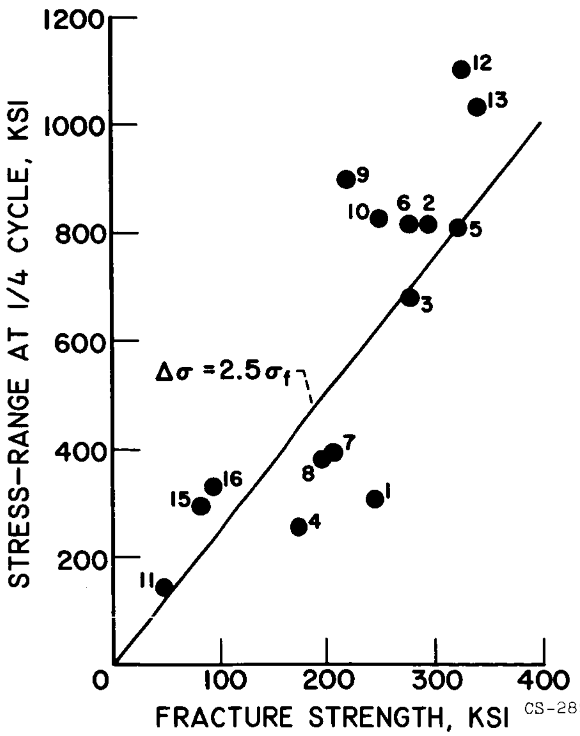


Figure 19. - Plastic strain range at life of 10 cycles versus ductility.



CS-28934

Figure 20. - Correlation of stress range at 10^5 cycles with ultimate strength. See Table I for material identification.



CS-28933

Figure 21. - Correlation of stress range at 1/4 cycle with fracture stress. See Table I for material identification.

E-2256

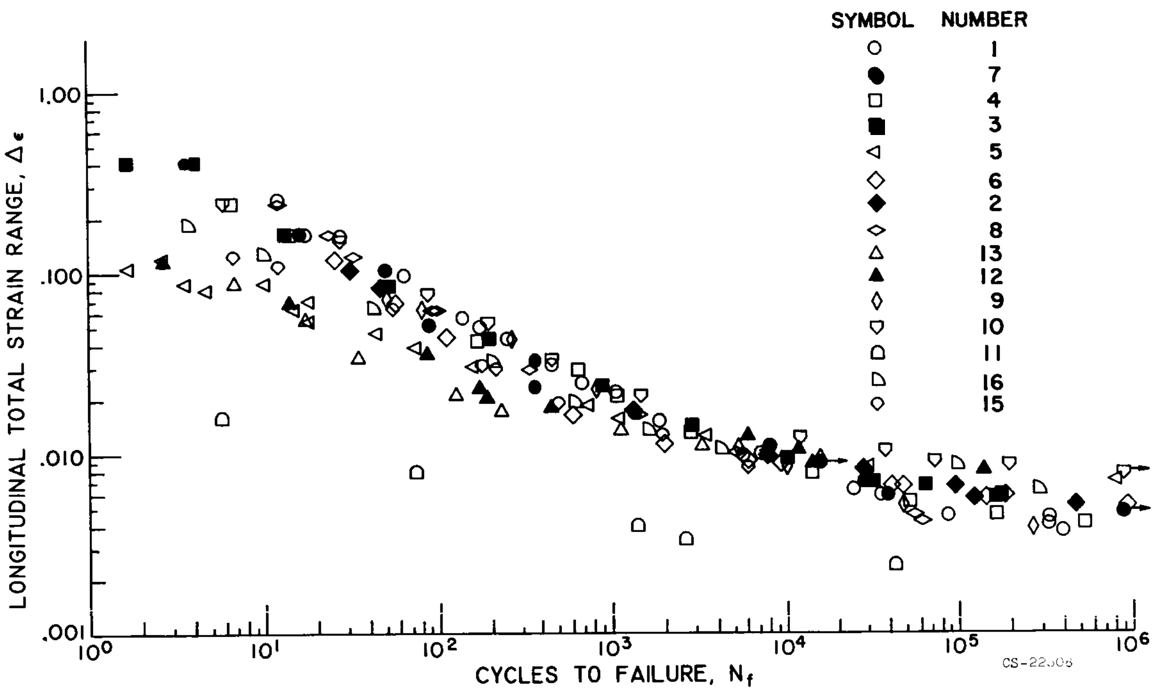
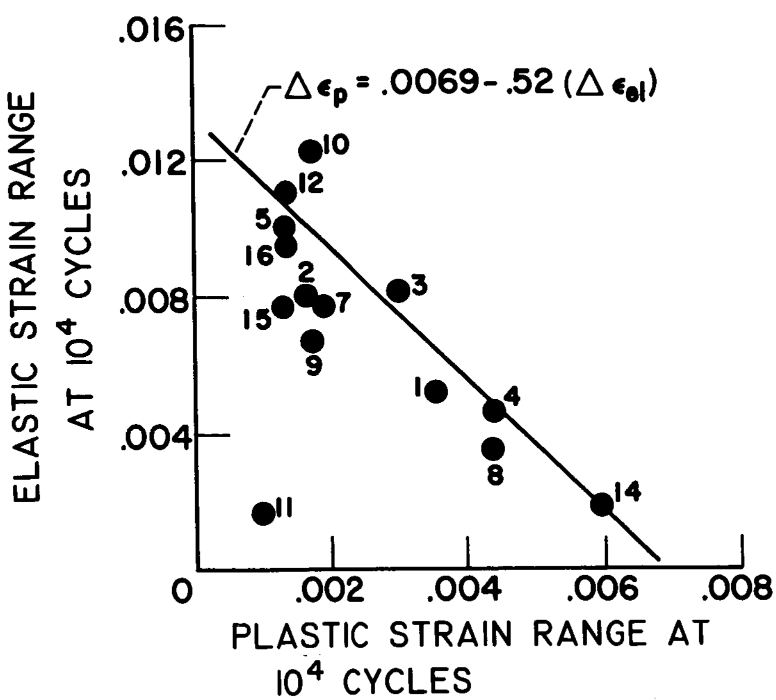


Figure 22. - Total strain versus cyclic life for all materials tested. See Table I for material identification.



CS-28936

Figure 23. - Correlation of elastic and plastic strain components at 10^4 cycles. See Table I for material identification.

E-2256

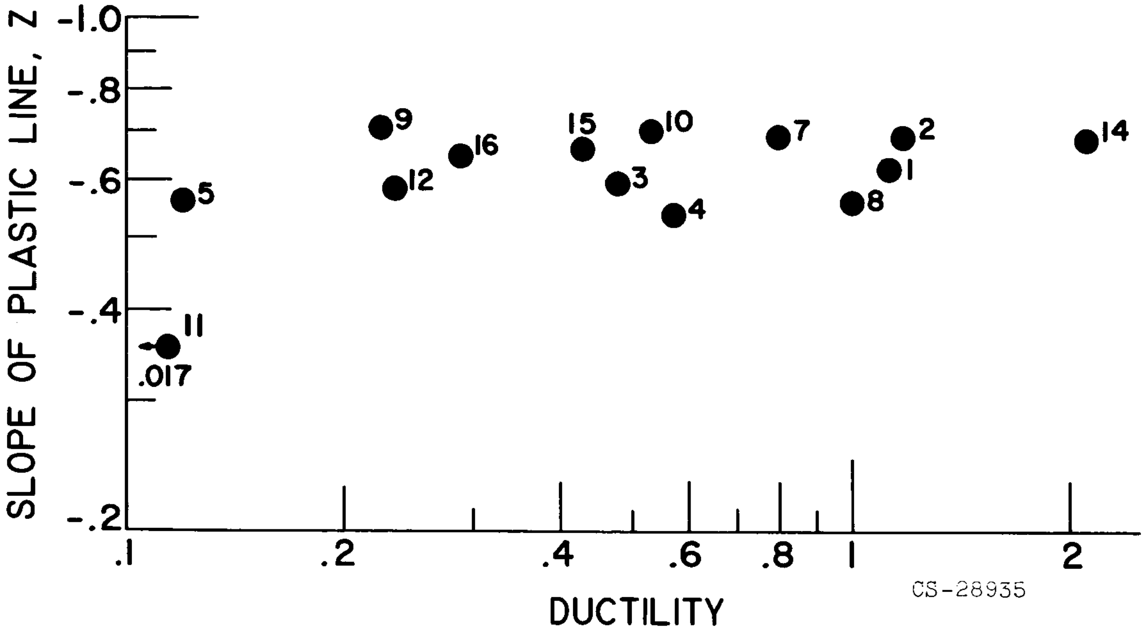


Figure 24. - Slope of plastic line versus ductility. See Table I for material identification.

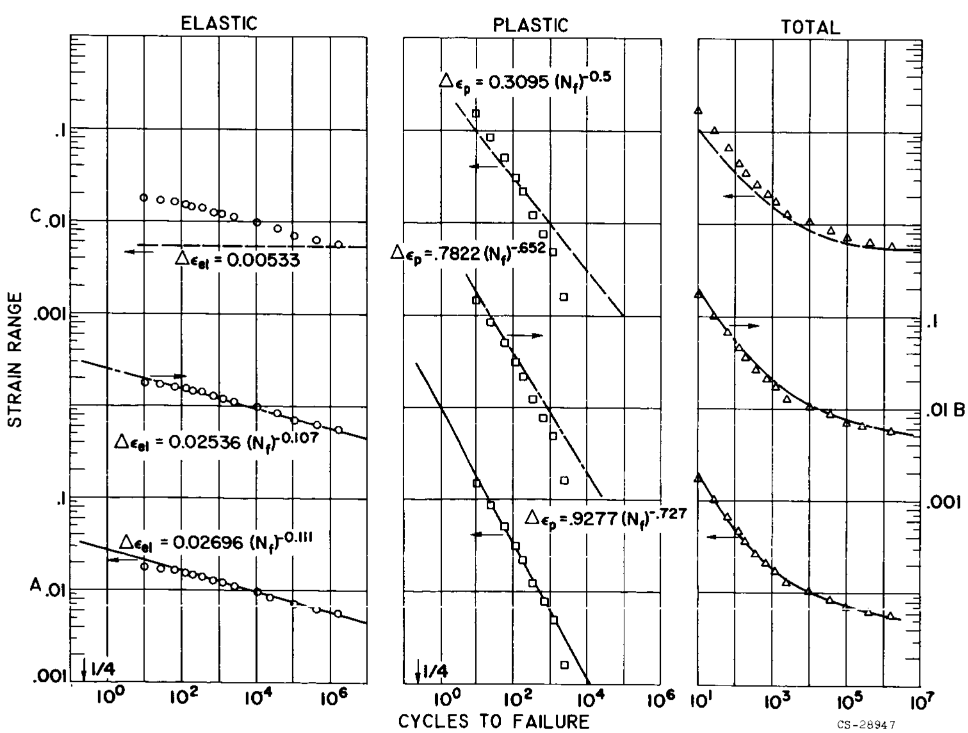


Figure 25. - Fatigue behavior of 4130 X-hard RC 48, material number 17.

E-2256

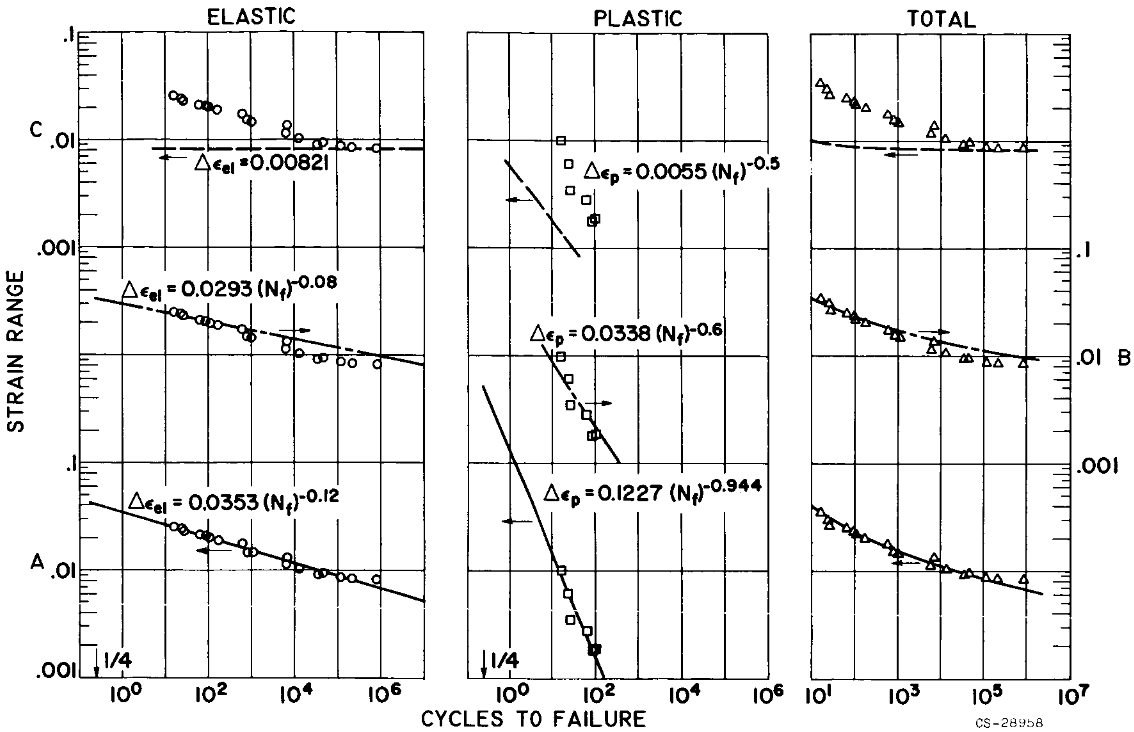


Figure 26. - Fatigue behavior of 52100 X-hard RC 62, material number 18.

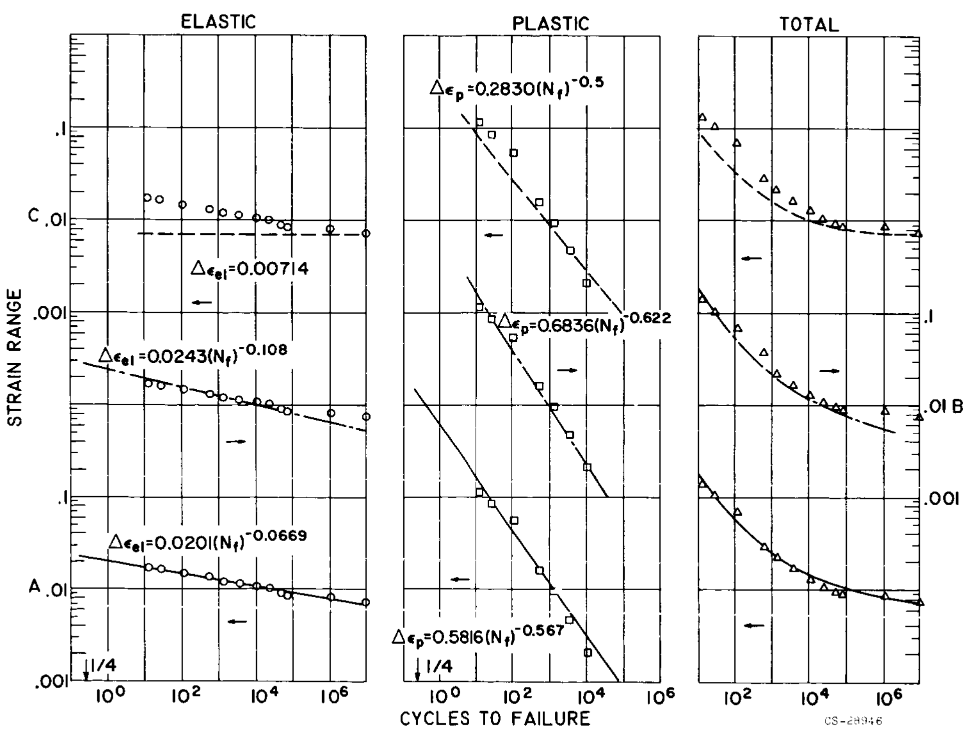


Figure 27. - Fatigue behavior of titanium 5AL-2 1/2 Sn, material number 19.

E-2256

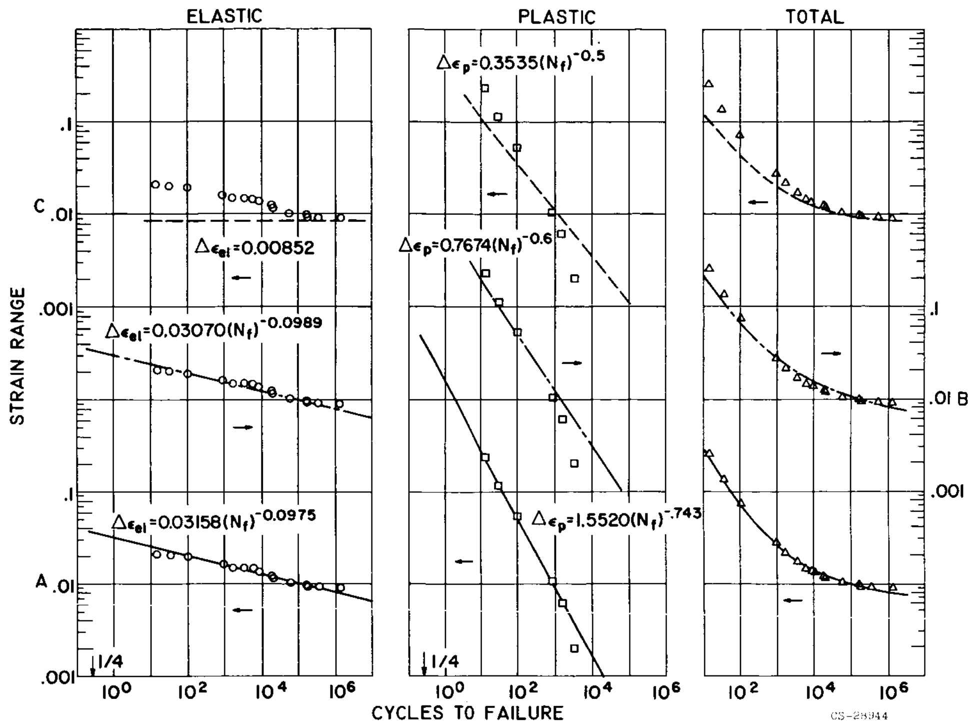


Figure 28. - Fatigue behavior of vascomax 300 CVM, material number 20.

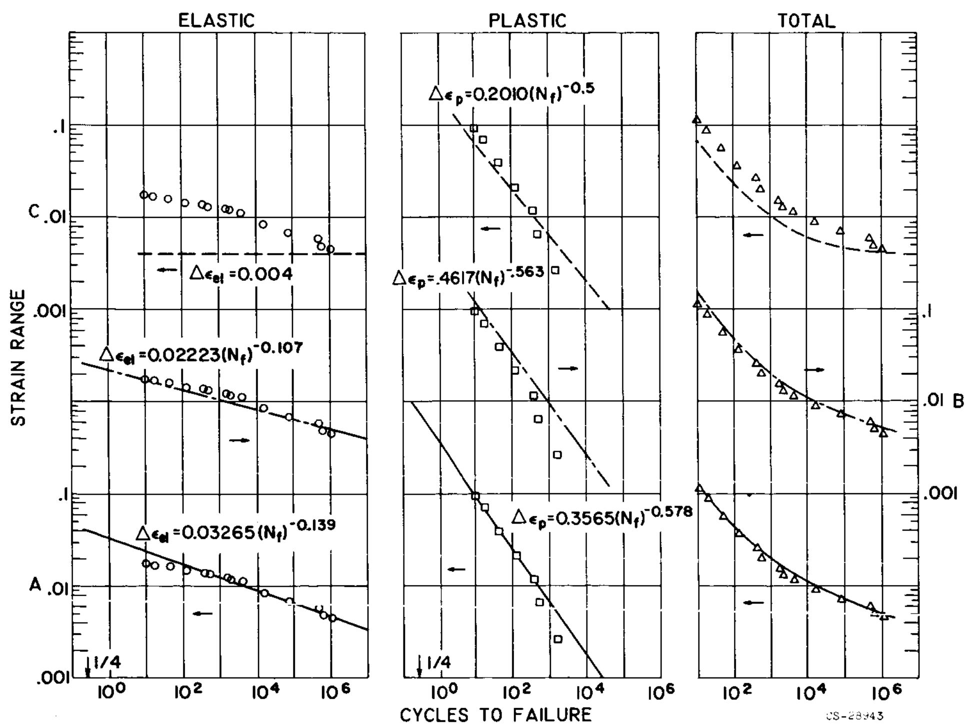


Figure 29. - Fatigue behavior of 2024 T4 aluminum, material number 21.

E-2256

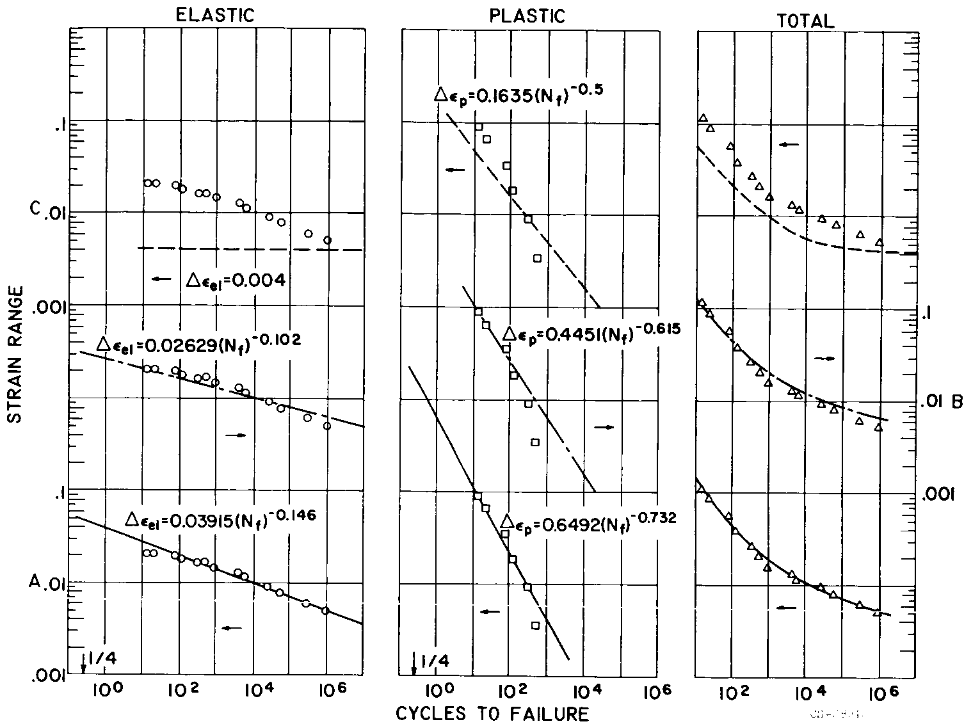


Figure 30. - Fatigue behavior of 7075 T6 aluminum, material number [illegible].

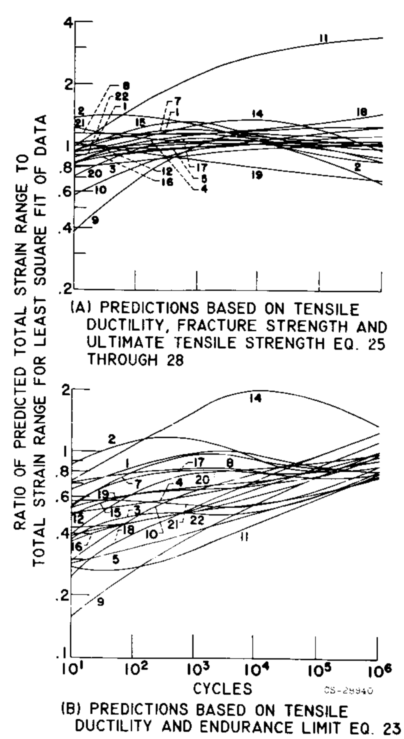


Figure 31. - Ratios of predicted to "least squares" total strain ranges versus cyclic lives.

E-2256

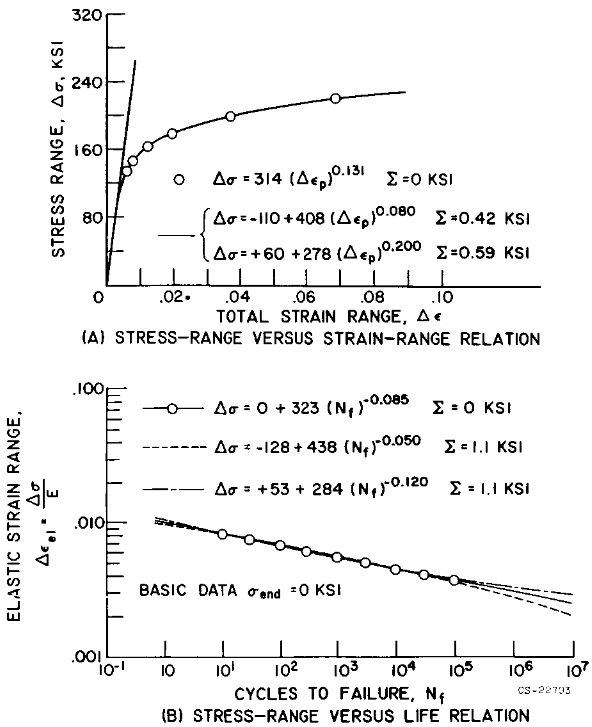


Figure 32. - Insensitivity of curves to constants in equations involving endurance limits.

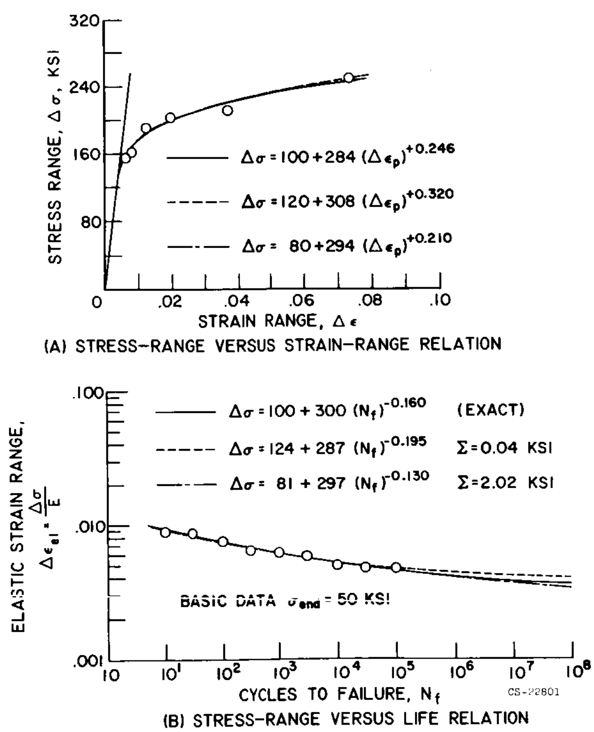


Figure 33. - Insensitivity of curves to constants in equations involving endurance limits.